

Flavors of Commonality: Learning in a Multiple Variant Environment

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Introduction

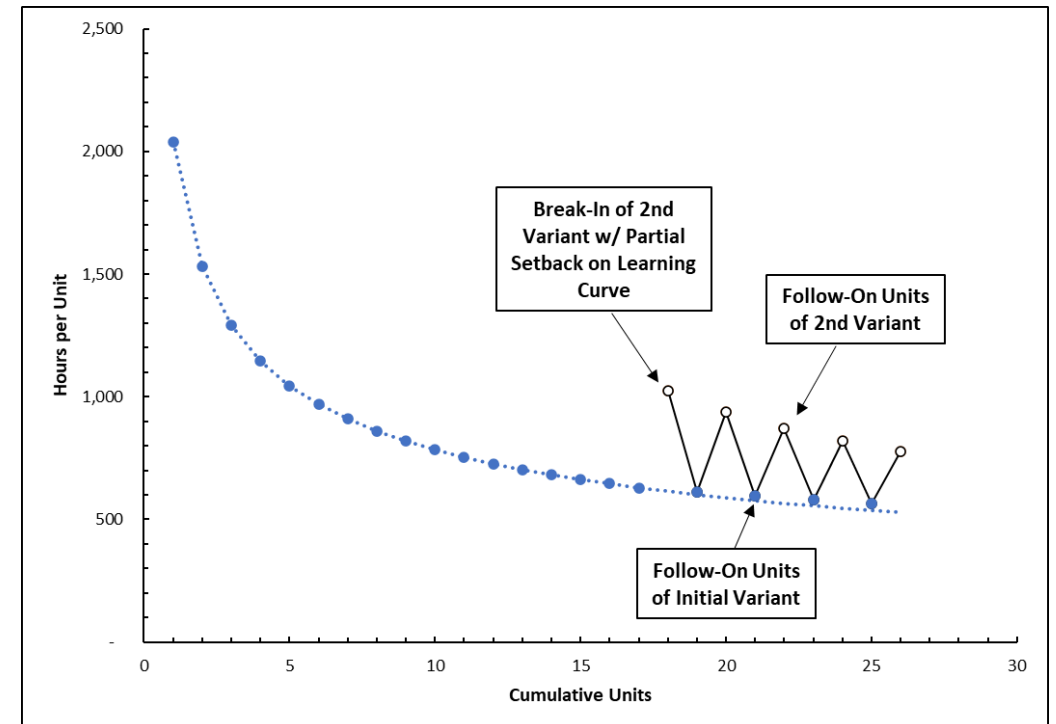
- **“Commonality” is the reuse of parts, designs, tools, engineering, and/or manufacturing processes between different models or variants.**
- **Frequently observed in the aircraft industry.**
- **Design commonality in commercial aircraft is promoted to reduce development costs, shorten the design cycle, and create greater market penetration.**
- **Military aircraft commonality is advanced as a strategy to save development, production and sustainment costs.**
 - **JAST program – the precursor to F-35 -- identified a potential EMD savings of 30-40% in airframe design, 40% savings in test, 30-40% savings in manufacturing and 60-70% savings in avionics for a common fighter program relative to three unique stand-alone programs.**

Examples of Commonality

- **One-seat (combat) and two-seat (trainer) configuration.**
 - **F-16C/D & F/A-18E/F fighters.**
- **Commercial jetliners with stretched or shortened fuselages.**
 - **Boeing 737 MAX comes in four versions (-7/-8/-9/-10) with same basic aircraft but different fuselage lengths and seating (138 to 204 passengers).**
- **Military aircraft supporting one military service but come in multiple configurations.**
 - **C-130J aircraft comes in standard cargo (J-30), tanker (KC-30), special operations (HC/MC), short body (J), weather reconnaissance (WC) or electronic variants (EC).**
- **Military aircraft supporting more than one military service.**
 - **F-35A/B/C, V-22, JSTARS E-8, JPATS T-6A, A-7, F-4.**

Commonality & Learning

- Learning curve theory assumes the same product is built repetitiously over multiple cycles resulting in a reduction of hours over time.
- If the product is not the same, however, we would expect that some learning loss from prior builds when the alternate configuration is built.
 - Similar to engineering design change where the configuration is altered, evidenced by a regression on the overall learning curve and higher hours per unit.
- Commonality really asks us, “How much learning transfer occurs between variants?”



What is “Common”?

- **Some suggested approaches:**
 - **Count number of common vs unique engineering drawings. (Garg, 1961)**
 - **Count number of common vs unique parts.**
 - **Sum the empty weight of common vs unique parts.**
 - **Sum the Industrial Engineering standard hours of common vs unique parts.**
 - **Engineering judgment based on the similarity or uniqueness of assembly processes and tooling.**
- **This is not an exhaustive list...Zhang (2019) lists no less than 7 other methods to assess commonality.**

What about parts that are similar but not common?

“Cousinality”

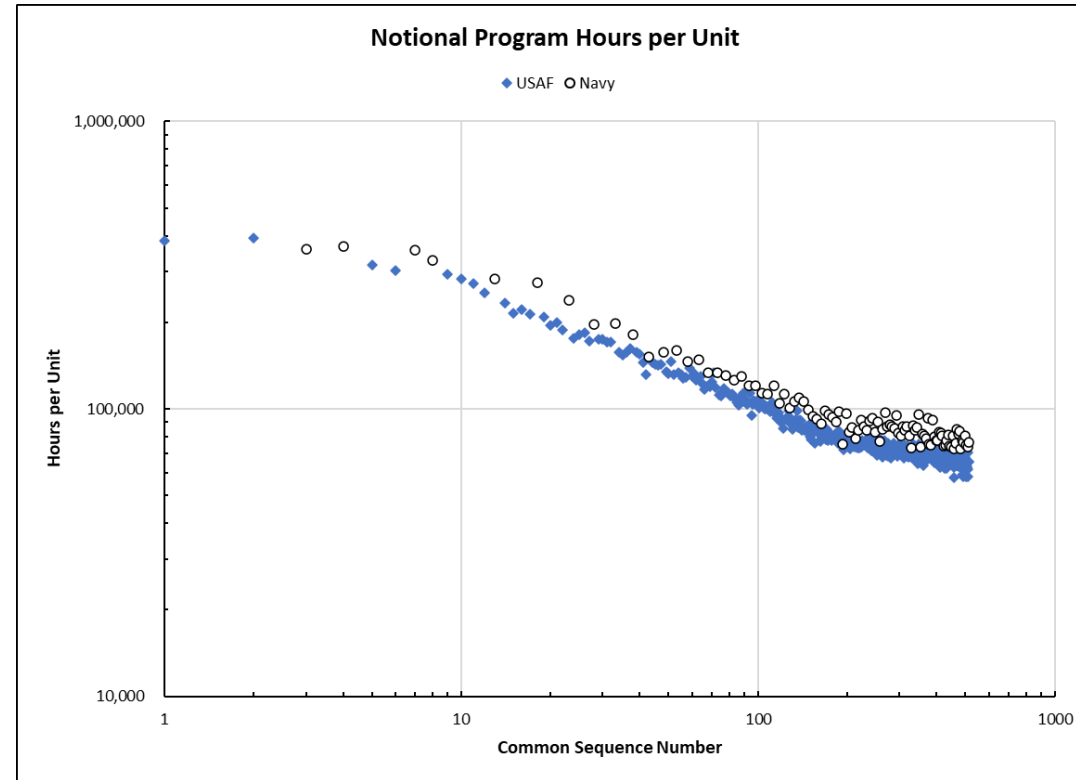
- **Parts or assemblies may be highly similar between two or more variants, but not *identical*.**
- **Expect similar parts should show some degree of learning transfer.**
- **JAST program created following definitions:**
 - **Common: Physically identical and interchangeable.**
 - **Cousin: Same material, function, and interfaces – similar internal geometry, e.g., bulkheads made of identical material, same external dimensions, yet different web thickness and number of penetrations). Made using common fabrication or assembly tooling.**
 - **Unique: Single variant application.**
- **For evaluation purposes, JAST treated a “cousin” part as 85% common & 15% unique.**

Notional Program

- **To illustrate how we might approach this problem but avoid compromising proprietary information, a dummy set of data has been constructed.**
- **Two variant aircraft program:**
 - **Eight EMD aircraft and 500+ production aircraft are built.**
 - **80% of aircraft built in an U.S. Air Force (USAF) configuration (Model A).**
 - **20% of aircraft built in a U.S. Navy (USN) configuration (Model B).**
 - **Assume 65% of the manufacturing effort is common between the USAF and USN versions, with the remaining 35% being unique to each variant.**
- **Using 5 different approaches, use the first 370 aircraft (EMD and Lots 1-11) to develop historical learning curve slopes.**
- **Apply historical learning curves to forecast the next 144 aircraft (Lots 12-14).**
- **Compare the forecast to the realized hours for those later aircraft.**

Notional Dataset

Total	Phase	Service	Variant	Adj Hours
1	EMD	USAF	A	384,354
2	EMD	USAF	A	392,722
3	EMD	Navy	B	359,041
4	EMD	Navy	B	366,820
5	EMD	USAF	A	316,530
6	EMD	USAF	A	303,031
7	EMD	Navy	B	355,896
8	EMD	Navy	B	329,786
9	Lot 1	USAF	A	294,270
10	Lot 1	USAF	A	283,824
	⋮			⋮
505	Lot 14	USAF	A	62,361
506	Lot 14	USAF	A	62,911
507	Lot 14	USAF	A	61,762
508	Lot 14	Navy	B	73,903
509	Lot 14	USAF	A	70,701
510	Lot 14	USAF	A	57,930
511	Lot 14	USAF	A	61,808
512	Lot 14	USAF	A	65,429
513	Lot 14	Navy	B	76,368
514	Lot 14	USAF	A	65,222



Data shows “S” curve shape seen on many historical programs

Commonality & Learning Curve - Approaches

- Jones (2019) suggests 4 different approaches to estimating commonality....I have added a fifth:
 - Ignore Differences (ID) – Assume a common learning curve and ignore any cost impact of multiple models.
 - Fixed Factors (FF) – Assume a common underlying curve and adjust for variant differences through a fixed factor or relationship between variants.
 - Total Separation (TS) – Assume each variant has a unique learning curve and that no learning transfer occurs between variants.
 - Partial Separation (PS) – Assume each variant has a unique learning curve but allow learning transfer between variants. **Added**
 - Proportional Representation (PR) – Assume a given combination of common or unique work has its own peculiar learning curve, but all of them share a common rate of learning.

Learning Curve Breaks

- **To develop historical learning, we will use a piecewise regression described in prior papers. (Johnstone, 2022)**

$$\ln y = \ln(\alpha_1 + \alpha_2) + (\beta_1 + \beta_2) \ln Dx$$

Where:

y = Manufacturing hours per unit (HPU)

α_1 = Y-intercept for leg #1, equal to theoretical first unit hours for leg #1

α_2 = Intercept adjustment for leg #2, such that $\alpha_1 + \alpha_2$ equals the Y-intercept for leg #2

β_1 = Rate of learning for leg #1, such that 2^{β_1} equals learning curve slope #1

β_2 = Rate of learning for leg #2, such that $2^{(\beta_1 - \beta_2)}$ equals learning curve for leg #2

- **...but slightly more complicated (three legs vs two in the 2022 JCAP article).**
- **Essentially, we will “break” the curve into different segments (EMD / early production / late production).**
- **But we will make different assumptions about rates of learning, counting cumulative learning, and how much learning transfer occurs.**

Ignore Differences (ID)

Cumulative units built for our regression

Log of cum units in leg #1

Log of cum units in leg #2

Log of cum units in leg #3

Dummy variable for leg #2

Dummy variable for leg #2

Common Sequence Number	Effective Sequence Number	Model	HPU	Curve Breakpoints		Dependent Variable	Independent Variables				
				T ₁	T ₂		LN(HPU)	β ₁	α ₂	β ₂	α ₃
1	1	A	384,354	9	151	12.86	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-
...
149	149	A	87,845	9	151	11.27	-	1	5.00	-	-
150	150	A	79,812	9	151	11.27	-	1	5.01	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04
...
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91

- Assume there is *no* difference between variants.
- Sometimes there may be different models but no observable cost difference.
 - L-1011 jetliner had multiple models (-1, -100, -200, -250, -500) but only -500 shortened model showed significant cost delta.

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	LN_HRS = 12.89 + (-0.09531) * BETA1 + (-0.4265) * BETA2 + (-0.1968) * BETA3 + 0.6366 * ALPHA2 + (-0.5584) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8913	0.0601		214.6371	0.0000	1.0000
BETA1	-0.0953	0.0406	-0.0553	-2.3484	0.0193	0.9807
BETA2	-0.4265	0.0092	-2.4114	-46.4237	0.0000	1.0000
BETA3	-0.1968	0.0200	-1.4561	-9.8498	0.0000	1.0000
ALPHA2	0.6366	0.0716	0.8415	8.8915	0.0000	1.0000
ALPHA3	-0.5584	0.1259	-0.7451	-4.4363	0.0000	1.0000

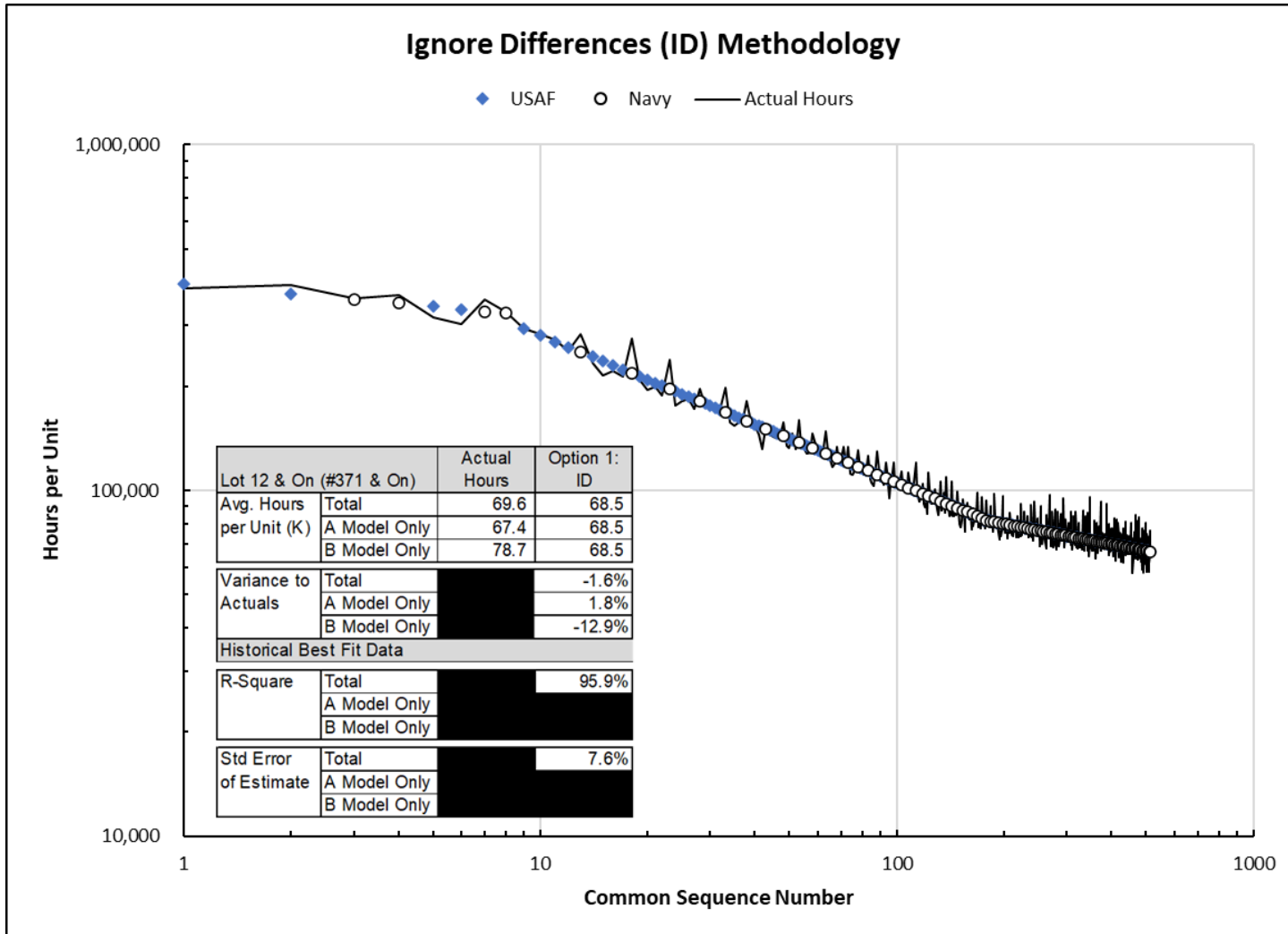
TFU - Leg 1	396,828
Slope - Leg 1	93.6%
Slope - Leg 2	74.4%
Slope - Leg 3	87.3%
TFU - Leg 2	750,051
TFU - Leg 3	227,039

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0755	95.85%	95.80%	0.9791	2.1439	95.72%



Ignore Differences (ID)



- Using our historical EMD-Lot 11 curve, forecast is within 1.6% of actual Lot 12-14 hours.
- But data shows a clear cost difference between USAF & USN variants.
- Model would not be appropriate except for very 'rough-cut' estimates.

Fixed Factors (FF)

If B model then equal to 1, otherwise 0

Common Sequence Number	Effective Sequence Number	Model	HPU	Curve Breakpoints		Dependent Variable	Independent Variables					B Model Dummy
				T ₁	T ₂		LN(HPU)	β ₁	α ₂	β ₂	α ₃	
1	1	A	384,354	9	151	12.86	-	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-	1
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-	1
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-	1
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-	1
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-	-
⋮												
149	149	A	87,845	9	151	11.38	-	1	5.00	-	-	-
150	150	A	79,812	9	151	11.29	-	1	5.01	-	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02	-
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02	-
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03	1
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04	-
⋮												
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90	-
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91	-
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91	1
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91	-
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91	-

- Assume a common underlying curve and adjust for variant differences through a fixed factor.
- We have made one change to the data....added a dummy variable for the B model.

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	$LN_HRS = 12.89 + (-0.1483) * BETA1 + (-0.4283) * BETA2 + (-0.1968) * BETA3 + 0.6199 * ALPHA2 + (-0.583) * ALPHA3 + 0.152 * B_MODEL$

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

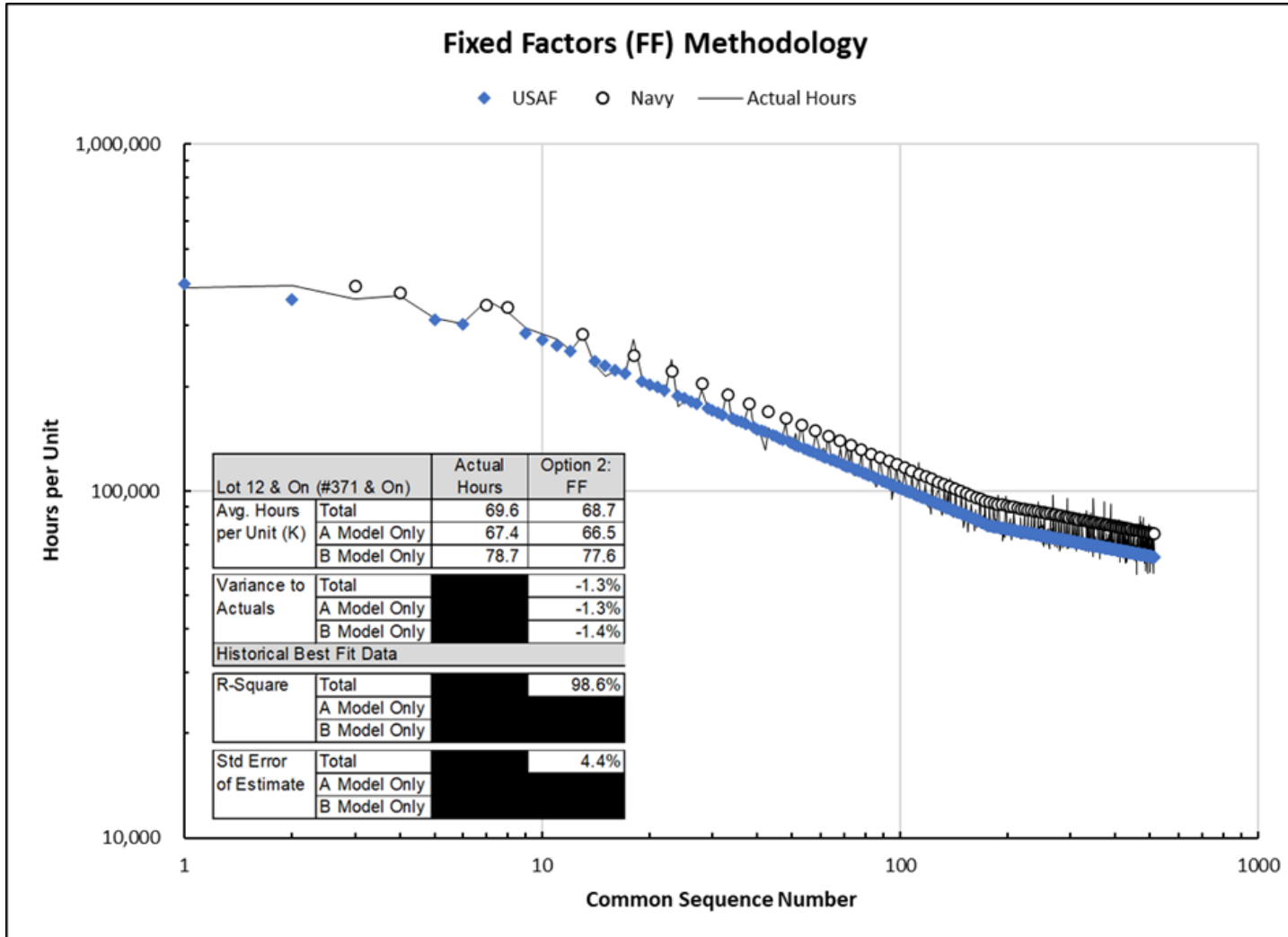
Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8855	0.0351		366.9231	0.0000	1.0000
BETA1	-0.1483	0.0238	-0.0870	-6.2282	0.0000	1.0000
BETA2	-0.4283	0.0054	-2.4215	-79.7236	0.0000	1.0000
BETA3	-0.1968	0.0117	-1.4562	-16.8467	0.0000	1.0000
ALPHA2	0.6199	0.0419	0.8193	14.8043	0.0000	1.0000
ALPHA3	-0.5830	0.0736	-0.7780	-7.9213	0.0000	1.0000
B_MODEL	0.1520	0.0057	0.1669	26.4904	0.0000	1.0000

TFU - Leg 1	394,563
Slope - Leg 1	90.2%
Slope - Leg 2	74.3%
Slope - Leg 3	87.2%
TFU - Leg 2	733,354
TFU - Leg 3	220,252
B Model Factor	1.164

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0442	98.59%	98.56%	0.9929	0.7453	98.51%

Fixed Factors (FF)



- Using our historical EMD-Lot 11 curve, forecast is within 1.3% of actual Lot 12-14 hours.
- More important, our A and B model forecasts are much closer to the realized actual hours.
- Assumes the relationship between A and B models is relatively constant over time.

Total Separation (TS)

Each variant has its own cumulative build number (no longer common)

Breakpoints for the legs have been modified as well

Common Sequence Number	Effective Sequence Number	Model	HPU	Breakpoints		Variable	Independent Variables				
				T ₁	T ₂		LN(HPU)	β ₁	α ₂	β ₂	α ₃
1	1	A	384,354	5	119	12.86	-	-	-	-	-
2	2	A	392,722	5	119	12.88	0.69	-	-	-	-
3	1	B	359,041	5	33	12.79	-	-	-	-	-
4	2	B	366,820	5	33	12.81	0.69	-	-	-	-
5	3	A	316,530	5	119	12.67	1.10	-	-	-	-
6	4	A	303,031	5	119	12.62	1.39	-	-	-	-
7	3	B	355,896	5	33	12.78	1.10	-	-	-	-
8	4	B	329,786	5	33	12.71	1.39	-	-	-	-
9	5	A	294,270	5	119	12.59	-	1	1.61	-	-
10	6	A	283,824	5	119	12.56	-	1	1.79	-	-
⋮											
149	117	A	87,845	5	119	11.38	-	1	4.76	-	-
150	118	A	79,812	5	119	11.29	-	1	4.77	-	-
151	119	A	78,318	5	119	11.27	-	-	-	1	4.78
152	120	A	81,745	5	119	11.31	-	-	-	1	4.79
153	33	B	94,523	5	33	11.46	-	-	-	1	3.50
154	121	A	86,816	5	119	11.37	-	-	-	1	4.80
⋮											
366	291	A	66,039	5	119	11.10	-	-	-	1	5.67
367	292	A	66,241	5	119	11.10	-	-	-	1	5.68
368	76	B	78,852	5	33	11.28	-	-	-	1	4.33
369	293	A	72,902	5	119	11.20	-	-	-	1	5.68
370	294	A	71,358	5	119	11.18	-	-	-	1	5.68

- Assume *no* learning transfer between variants & each version experiences a different rate of learning.

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	234
Equation in Unit Space:	LN_HRS = 12.9 + (-0.1862) * BETA1 + (-0.3977) * BETA2 + (-0.209) * BETA3 + 0.3635 * ALPHA2 + (-0.5838) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.9047	0.0358		360.3412	0.0000	1.0000
BETA1	-0.1862	0.0377	-0.0589	-4.9395	0.0000	1.0000
BETA2	-0.3977	0.0049	-2.2356	-80.9902	0.0000	1.0000
BETA3	-0.2090	0.0115	-1.5626	-18.1867	0.0000	1.0000
ALPHA2	0.3635	0.0408	0.5084	8.9133	0.0000	1.0000
ALPHA3	-0.5838	0.0707	-0.8213	-8.2568	0.0000	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0393	98.76%	98.73%	0.9938	0.5403	98.49%

USAF	
TFU - Leg 1	402,196
Slope - Leg 1	87.9%
Slope - Leg 2	75.9%
Slope - Leg 3	86.5%
TFU - Leg 2	578,528
TFU - Leg 3	224,336

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	76
Equation in Unit Space:	LN_HRS = 12.81 + (-0.05086) * BETA1 + (-0.5459) * BETA2 + (-0.1468) * BETA3 + 0.5661 * ALPHA2 + (-0.8621) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

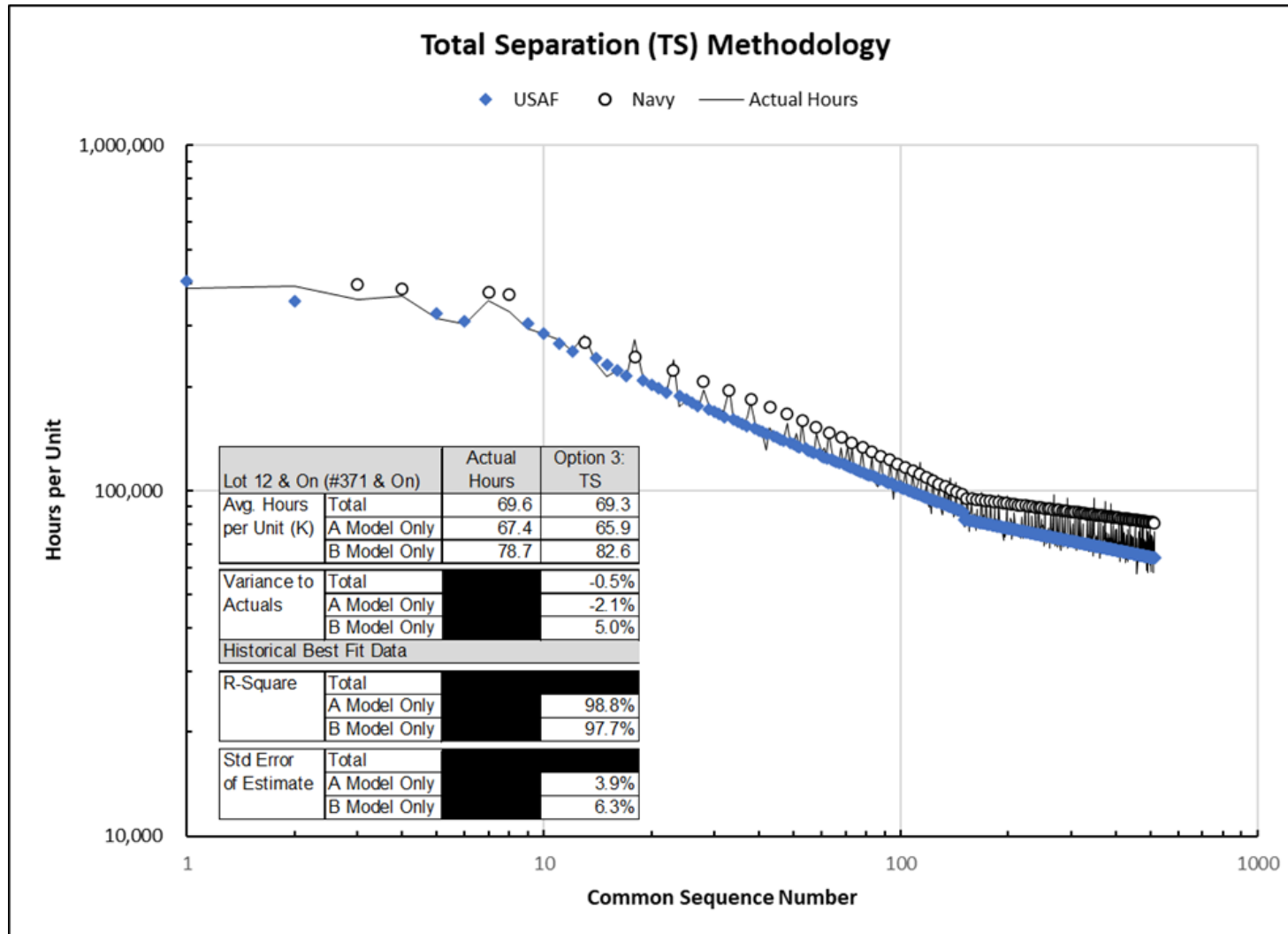
Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8135	0.0579		221.4631	0.0000	1.0000
BETA1	-0.0509	0.0609	-0.0272	-0.8351	0.4065	0.5935
BETA2	-0.5459	0.0231	-1.8901	-23.6604	0.0000	1.0000
BETA3	-0.1468	0.0394	-0.7215	-3.7246	0.0004	0.9996
ALPHA2	0.5661	0.0875	0.6818	6.4670	0.0000	1.0000
ALPHA3	-0.8621	0.1671	-1.0629	-5.1601	0.0000	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0634	97.63%	97.53%	0.9884	0.3430	97.13%

USN	
TFU - Leg 1	367,146
Slope - Leg 1	96.5%
Slope - Leg 2	68.5%
Slope - Leg 3	90.3%
TFU - Leg 2	646,678
TFU - Leg 3	155,033

Total Separation (TS)



- Using our historical EMD-Lot 11 curve, forecast is within 0.5% of actual Lot 12-14 hours.
- However, the forecast errors are larger (2-5%) at the individual variant.
- TS approach would be best suited where different models are built on separate production lines, e.g. Eurofighter where final assembly occurs in 4 separate countries.

Partial Separation (PS)

Returned to a common unit count for all variants

Returned to the prior curve breakpoints

Common Sequence Number	Effective Sequence Number	Model	HPU	Curve Breakpoints		Dependent Variable	Independent Variables				
				T ₁	T ₂	LN(HPU)	β ₁	α ₂	β ₂	α ₃	β ₃
1	1	A	384,354	9	151	12.86	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-
⋮											
149	149	A	87,845	9	151	11.38	-	1	5.00	-	-
150	150	A	79,812	9	151	11.29	-	1	5.01	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04
⋮											
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	234
Equation in Unit Space:	$LN_HRS = 12.91 + (-0.1452) * BETA1 + (-0.4264) * BETA2 + (-0.211) * BETA3 + 0.5924 * ALPHA2 + (-0.5247) * ALPHA3$

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.9053	0.0335		384.8329	0.0000	1.0000
BETA1	-0.1452	0.0268	-0.0606	-5.4261	0.0000	1.0000
BETA2	-0.4264	0.0052	-2.5485	-82.2160	0.0000	1.0000
BETA3	-0.2110	0.0114	-1.8463	-18.4469	0.0000	1.0000
ALPHA2	0.5924	0.0401	0.8284	14.7761	0.0000	1.0000
ALPHA3	-0.5247	0.0717	-0.7383	-7.3223	0.0000	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0387	98.73%	98.77%	0.9939	0.4832	98.65%

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	76
Equation in Unit Space:	$LN_HRS = 12.89 + (-0.07207) * BETA1 + (-0.437) * BETA2 + (-0.1398) * BETA3 + 0.8028 * ALPHA2 + (-0.7484) * ALPHA3$

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8904	0.1263		102.0407	0.0000	1.0000
BETA1	-0.0721	0.0754	-0.0675	-0.9563	0.3422	0.6578
BETA2	-0.4370	0.0175	-2.2601	-24.9591	0.0000	1.0000
BETA3	-0.1398	0.0358	-0.9557	-3.9088	0.0002	0.9998
ALPHA2	0.8028	0.1467	0.9670	5.4710	0.0000	1.0000
ALPHA3	-0.7484	0.2349	-0.9227	-3.1861	0.0021	0.9379

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0605	97.90%	97.75%	0.9894	0.2972	97.56%

USAF

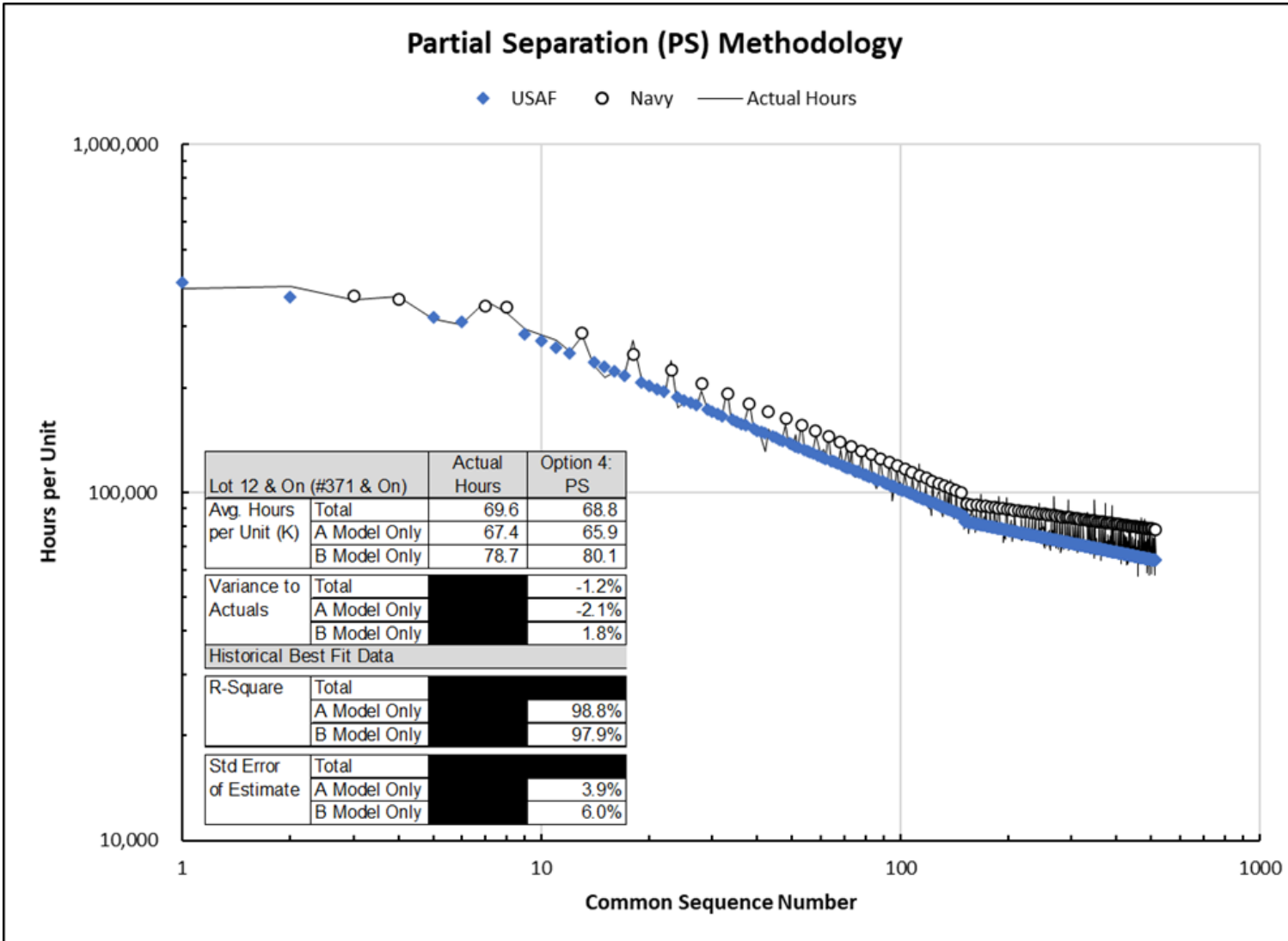
TFU - Leg 1	402,459
Slope - Leg 1	90.4%
Slope - Leg 2	74.4%
Slope - Leg 3	86.4%
TFU - Leg 2	727,760
TFU - Leg 3	238,136

USN

TFU - Leg 1	396,487
Slope - Leg 1	95.1%
Slope - Leg 2	73.9%
Slope - Leg 3	90.8%
TFU - Leg 2	884,908
TFU - Leg 3	167,584

- Similar to TS that each version experiences a different rate of learning, but we assume learning transfer exists between variants.

Partial Separation (PS)



- Using our historical EMD-Lot 11 curve, forecast is within 1.2% of actual Lot 12-14 hours.
- Forecast error at individual variant is reduced from TS method (2%).
- Does it make sense there should be different rates of learning?
 - ~80% of cost improvement attributed to factors besides operator learning. (Jefferson, 1981)
 - Depends on how personnel, tools, supply chain, manufacturing methods & investment strategies are managed across the variants.

Proportional Representation (PR)

- PR method breaks down the different combinations of common versus unique work.
- To demonstrate how complex commonality can get – and how PR might prove advantageous -- let's briefly introduce a C model into the discussion.
- With three variants (A, B and C models), there are seven (7) possible combinations of common and unique work.
 - ABC Common
 - AB Common
 - AC Common
 - BC Common
 - A Unique
 - B Unique
 - C Unique
- PR method assumes a given combination of common or unique work has its own peculiar learning curve, but all of them share a common rate of learning.

The “Seven Flavors of Commonality”

Number of combinations calculated as $2^x - 1$

Variants	Number
2	3
3	7
4	15
5	31
6	63

Proportional Representation (PR)

Model	Percent Common to Each Model							Total
	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	
A	50%	15%	10%		25%			100%
B	50%	15%		5%		30%		100%
C	50%		10%	5%			35%	100%

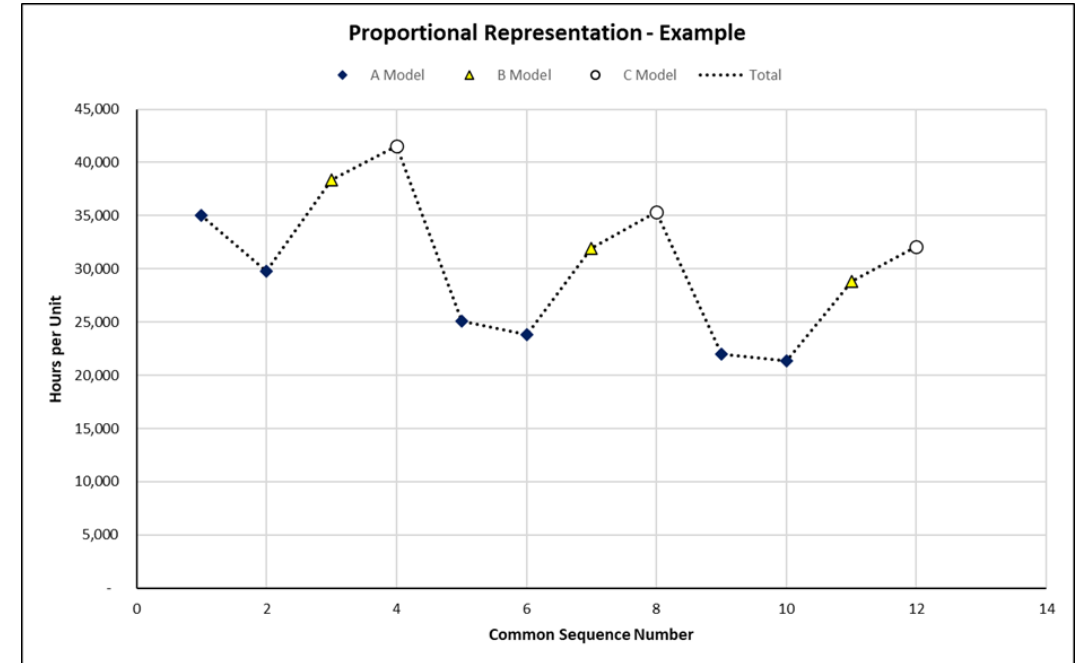
- TFU hours are broken by “flavor” and run down 85% slope but with different sequence numbers.

Learning Curve Slope	85%
Learning Beta	-0.23447

Work Content Split	A	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Variant Total
A		50%	15%	10%		25%			100%
B		50%	15%		5%		30%		100%
C		50%		10%	5%			35%	100%

T-1 Hours	A	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Totals
A	17,500	5,250	3,500	-	8,750	-	-	-	35,000
B	22,500	6,750	-	2,250	-	13,500	-	-	45,000
C	25,000	-	5,000	2,500	-	-	17,500	-	50,000

Model	Common/Unique Build Sequence Number							Hours per Unit							
	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Totals
A	1	1	1		1			17,500	5,250	3,500		8,750			35,000
A	2	2	2		2			14,875	4,463	2,975		7,438			29,750
B	3	3		1		1		17,391	5,217		2,250		13,500		38,358
C	4		3	2				18,063		3,865	2,125			17,500	41,552
A	5	4	4		3			11,999	3,793	2,529		6,763			25,084
A	6	5	5		4			11,497	3,600	2,400		6,322			23,819
B	7	6		3		2		14,257	4,435		1,739		11,475		31,906
C	8		6	4				15,353		3,285	1,806			14,875	35,319
A	9	7	7		5			10,454	3,327	2,218		6,000			21,999
A	10	8	8		6			10,199	3,224	2,149		5,749			21,322
B	11	9		5		3		12,824	4,032		1,543		10,434		28,833
C	12		9	6				13,961		2,987	1,642			13,526	32,116



- Produces “sawtooth” pattern we would expect to see.

Proportional Representation (PR)

Percent Common to Each Model								
Model	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Total
A	50%	15%	10%		25%			100%
B	50%	15%		5%		30%		100%
C	50%		10%	5%			35%	100%



Commonality Matrix			
Model	A Credit	B Credit	C Credit
A	100%	65%	60%
B	65%	100%	55%
C	60%	55%	100%

For Every A Model Built:

Each A Model: A Receives 100% Learning Credit

Each B Model: ABC Common + AB Common = 50% + 15% = A Receives 65% Credit

Each C Model: ABC Common + AC Common = 50% + 10% = A Receives 60% Credit



Model	Cumulative Unit Count			Effective Unit
	A	B	C	
A	1	0	0	1.00
A	2	0	0	2.00
B	2	1	0	2.30
C	2	1	1	2.75
A	3	1	1	4.25
A	4	1	1	5.25
B	4	2	1	5.15
C	4	2	2	5.50
A	5	2	2	7.50
A	6	2	2	8.50
B	6	3	2	8.00
C	6	3	3	8.25

(3 A's x 100%) + (1 B x 65%) + (1 C x 60%) = Effective Unit 4.25

- However, this can get computationally difficult ... and very hard to calculate historical learning.

- Fortunately, we can use commonality matrix & calculate a single “effective sequence” number that incorporates different commonality by “flavor.”

Proportional Representation (PR)

Incorporates the sequence calculation from prior page

B Model dummy variable captures difference in standard work content (vs learning)

Common Sequence Number	Effective Sequence Number	Model	HPU	Curve Breakpoints		Dependent Variable	Independent Variables					B Model Dummy
				T ₁	T ₂		LN(HPU)	β ₁	α ₂	β ₂	α ₃	
1	1.0	A	384,354	7.6	139.8	12.86	-	-	-	-	-	-
2	2.0	A	392,722	7.6	139.8	12.88	0.69	-	-	-	-	-
3	2.3	B	359,041	7.6	111.0	12.79	0.83	-	-	-	-	-
4	3.3	B	366,820	7.6	111.0	12.81	1.19	-	-	-	-	1
5	4.3	A	316,530	7.6	139.8	12.67	1.46	-	-	-	-	-
6	5.3	A	303,031	7.6	139.8	12.62	1.67	-	-	-	-	-
7	5.6	B	355,896	7.6	111.0	12.78	1.72	-	-	-	-	1
8	6.6	B	329,786	7.6	111.0	12.71	1.89	-	-	-	-	1
9	7.6	A	294,270	7.6	139.8	12.59	-	1	2.03	-	-	-
10	8.6	A	283,824	7.6	139.8	12.56	-	1	2.15	-	-	-
⋮												
149	137.8	A	87,845	7.6	139.8	11.38	-	1	4.93	-	-	-
150	138.8	A	79,812	7.6	139.8	11.29	-	1	4.93	-	-	-
151	139.8	A	78,318	7.6	139.8	11.27	-	-	-	1	4.94	-
152	140.8	A	81,745	7.6	139.8	11.31	-	-	-	1	4.95	-
153	111.0	B	94,523	7.6	111.0	11.46	-	-	-	1	4.71	1
154	142.5	A	86,816	7.6	139.8	11.37	-	-	-	1	4.96	-
⋮												
366	339.8	A	66,039	7.6	139.8	11.10	-	-	-	1	5.83	-
367	340.8	A	66,241	7.6	139.8	11.10	-	-	-	1	5.83	-
368	265.8	B	78,852	7.6	111.0	11.28	-	-	-	1	5.58	1
369	342.4	A	72,902	7.6	139.8	11.20	-	-	-	1	5.84	-
370	343.4	A	71,358	7.6	139.8	11.18	-	-	-	1	5.84	-

We'll return back to our two model (A & B model) case....

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	LN_HRS = 12.88 + (-0.1342) * BETA1 + (-0.4215) * BETA2 + (-0.2076) * BETA3 + 0.554 * ALPHA2 + (-0.5313) * ALPHA3 + 0.08574 * B_MODEL

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8807	0.0355		363.2449	0.0000	1.0000
BETA1	-0.1342	0.0268	-0.0704	-5.0024	0.0000	1.0000
BETA2	-0.4215	0.0054	-2.3133	-77.6938	0.0000	1.0000
BETA3	-0.2076	0.0116	-1.5020	-17.8864	0.0000	1.0000
ALPHA2	0.5540	0.0419	0.7322	13.2168	0.0000	1.0000
ALPHA3	-0.5313	0.0722	-0.7089	-7.3544	0.0000	1.0000
B_MODEL	0.0857	0.0061	0.0941	13.9869	0.0000	1.0000

TFU - Leg 1	392,667
Slope - Leg 1	91.1%
Slope - Leg 2	74.7%
Slope - Leg 3	86.6%
TFU - Leg 2	683,295
TFU - Leg 3	230,835
B Model Factor	1.090

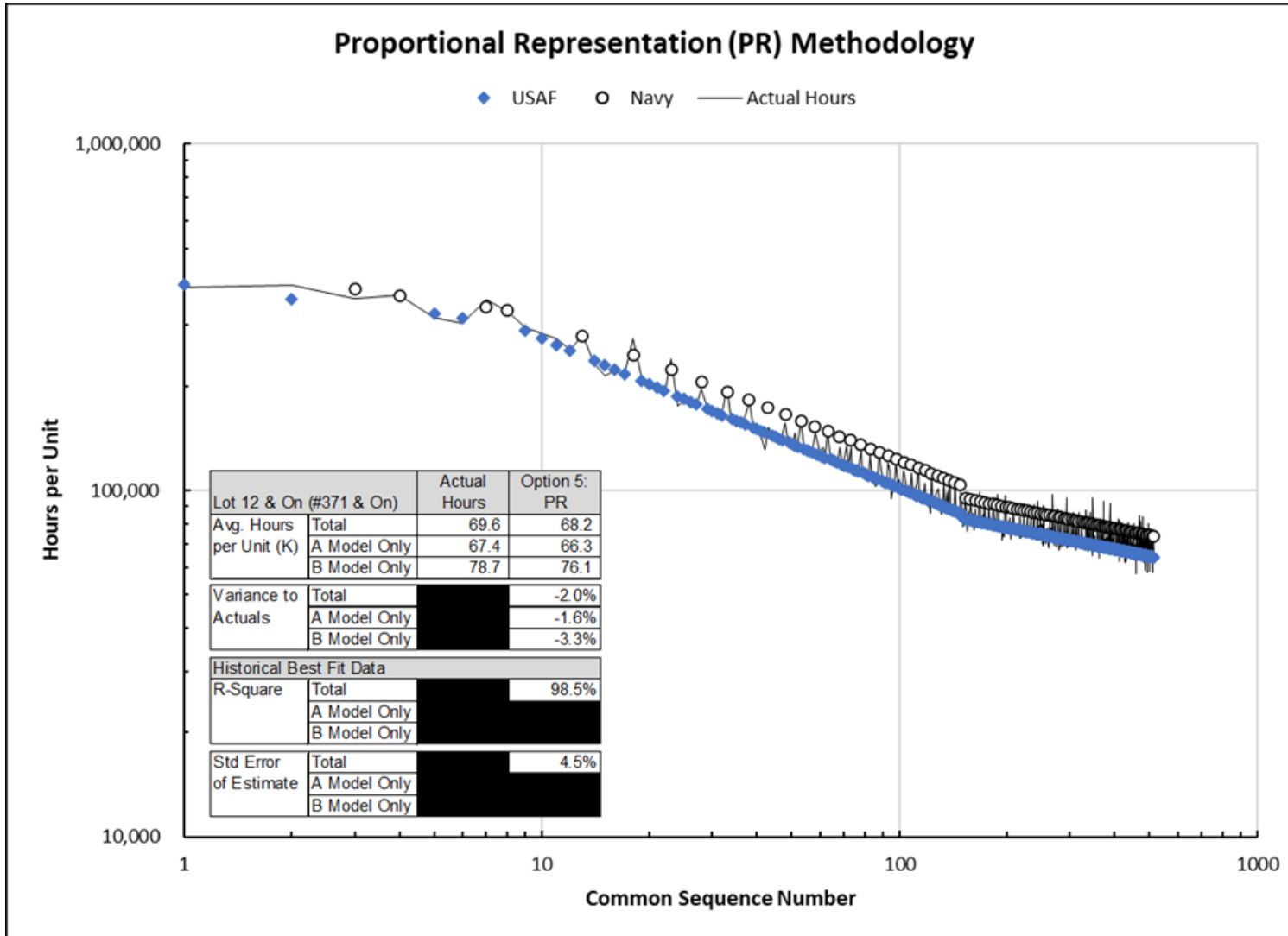
Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef	PRESS	R-Squared (Predicted)
0.0453	98.52%	98.49%	0.9925	0.7789	98.44%

- Common rate of learning, but each variant is on a different position on the learning curve at any given point.



Proportional Representation (PR)



- Using our historical EMD-Lot 11 curve, forecast is within 2% of actual Lot 12-14 hours.
- Forecast error at individual variant is reduced from TS method (2-3%).

Comparison of Results

Lot 12 & On (#371 & On)		Actual Hours	Option 1: ID	Option 2: FF	Option 3: TS	Option 4: PS	Option 5: PR
Avg. Hours per Unit (K)	Total	69.6	68.5	68.7	69.3	68.8	68.2
	A Model Only	67.4	68.5	66.5	65.9	65.9	66.3
	B Model Only	78.7	68.5	77.6	82.6	80.1	76.1
Variance to Actuals	Total		-1.6%	-1.3%	-0.5%	-1.2%	-2.0%
	A Model Only		1.8%	-1.3%	-2.1%	-2.1%	-1.6%
	B Model Only		-12.9%	-1.4%	5.0%	1.8%	-3.3%
Historical Best Fit Data							
R-Square	Total		95.9%	98.6%			98.5%
	A Model Only				98.7%	98.8%	
	B Model Only				97.7%	97.9%	
Std Error of Estimate	Total		7.6%	4.4%			4.5%
	A Model Only				3.9%	3.9%	
	B Model Only				6.3%	6.0%	

Legend:
 ID – Ignore Differences
 FF – Fixed Factors
 TS – Total Separation
 PS – Partial Separation
 PR – Proportional Representation

- **Performance of FF is not surprising – the notional data was generated using FF assumptions before introducing a random error to provide a realistic spread of values (we loaded the dice!)**
- **Had we generated the data using different premises, another method would probably produce a better forecast.**
- **Goal is not to prove one method is always superior to the others....The particulars of a program and its build circumstances will dictate which method is the preferred estimating approach.**

When Do I Use Which Method?

Methodology	More Appropriate If:	Less Appropriate If:
Ignore Differences ((D)	<ul style="list-style-type: none"> There is little or no cost difference between variants. 	<ul style="list-style-type: none"> Significant differences in work content exist between variants.
Fixed Factors (FF)	<ul style="list-style-type: none"> Significant amount of work is common or similar and the probability of learning transfer between variants is high. The cost variance between models is expected to be a fixed ratio in the future, e.g., B models are 10% more costly than A models. 	<ul style="list-style-type: none"> If component or subcomponent is variant-unique (TS may be more appropriate for that item).
Total Separation (TS)	<ul style="list-style-type: none"> Individual models are produced in different locations or on unique production lines, and the probability of learning transfer between variants is low. A component or subcomponent is variant-unique (FF, PS or PR may be used for the other, more common build areas). 	<ul style="list-style-type: none"> Models are built in the same location and/or same production line with work crews being cycled between models.

Methodology	More Appropriate If:	Less Appropriate If:
Partial Separation (PS)	<ul style="list-style-type: none"> Significant degree of common or similar work, but reason to believe each variant has a unique rate of learning. 	<ul style="list-style-type: none"> If the elements of learning that are common or similar between variants are high contributors to cost improvement, causing the rate of learning between variants to be roughly equal.
Proportional Representation (PR)	<ul style="list-style-type: none"> Significant amount of work is common or similar and the probability of learning transfer between variants is high. A fixed cost ratio between models cannot be established from actual cost history, or the relationship of one variant to another is expected to be different in the future. 	<ul style="list-style-type: none"> No suitable <i>a priori</i> methodology exists for determining the percentage of common vs unique work.

- **There are no hard and fast rules when to apply one methodology over another, but these are some guidelines that suggest when one approach might work better than another.**

Conclusions

- **Commonality is the reuse of parts & designs to reduce development & production costs.**
- **From a learning curve perspective, commonality asks: “How much learning transfer will occur between models or variants?”**
- **Built a notional program and applied 5 different approaches to estimating commonality costs/benefits.**
- **Approaches vary regarding assumptions on rates of learning, calculation of cumulative units, degrees of learning transfer.**
- **No one methodology is inherently superior to the others....which method we should use will depend on the particular circumstances we are estimating.**

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