Flavors of Commonality: Learning in a Multiple Variant Environment

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Abstract (75-word limit): Commonality – the reuse of parts, designs and tools across multiple aircraft models -- is a popular strategy to reduce program costs in commercial and military applications. But its use poses unique challenges to learning curve practitioners. This paper examines five approaches to estimating multiple variant programs using different learning curve techniques. A notional dataset is created, and the accuracy of each method is measured to highlight the advantages and disadvantages of each.

Key words: Learning Curves, Manufacturing, Commonality, Methods, Modeling, Labor

Introduction

Commonality refers to the reuse of parts, designs, tools, engineering, and/or manufacturing processes between different models or variants of a product. As a design and build strategy, commonality is frequently observed in both commercial and military aircraft. In the commercial world, design commonality is promoted to reduce development costs, shorten the design cycle, and create greater market penetration. (Zhang, 2019) Similarly, in military usage commonality is advanced as a strategy to save development, production and sustainment costs. The Joint Advanced Strike Technology (JAST) program identified a potential Engineering and Manufacturing Development (EMD) savings of 30-40% in airframe design, 40% savings in test, 30-40% savings in manufacturing and 60-70% savings in avionics for a common fighter program relative to three unique stand-alone programs.[1] (*JAST Commonality Study*, 1996).

Commonality can appear in various forms and degrees, including:

- Fighter aircraft which come in a one-seat (combat) or two-seat (trainer) configuration where the cockpit is the only distinguishable structural difference. The F-16C/D and the F/A-18E/F fighters are current examples.
- Commercial jetliners where fuselage lengths are stretched or shortened from a baseline configuration, allowing airlines to choose aircraft with greater or lesser seats to support a particular market. The Boeing 737 MAX comes in four versions (-7/-8/-9/-10) with the same basic aircraft but fuselage length ranging from 116 to 143 feet with seating ranging from 138 to 204 passengers. (*About the Boeing 737 MAX*, n.d.)
- Military aircraft designed to support a single military service but come in multiple configurations. For example, the C-130J aircraft comes in a standard cargo model (J-30), a tanker version (KC-30), special operations version (HC/MC), short body (J), weather reconnaissance (WC) or electronic variant (EC).
- Military aircraft designed to support more than one military service. Examples include the F-4 fighter, A-7 attack aircraft, JPATS T-6A, JSTARS E-8 and V-22. (Lorell, 2013) The most complex example is the F-35 fighter, whose A model (conventional takeoff and landing), B model (short takeoff and vertical landing), and C model (carrier variant) have sold over 1,000 aircraft to US and international services.

What impact might we expect to see on build hours? Learning curve theory assumes the same product is built repetitiously over multiple cycles resulting in a reduction of hours over time. If the product is not the same, however, we would expect that some learning loss from prior builds when the alternate configuration is built. This is like an engineering design change where there is the configuration is altered, resulting in some degree of lost learning, evidenced by a regression on the overall learning curve and higher hours per unit. (Johnstone, 2023) But building multiple models or variants of an aircraft has additional complexities over and above an engineering design change. For one thing, the impact of an individual engineering design change is time-limited: after the initial learning loss, there is a relatively

[1] The idea that commonality results in cost savings is not universally accepted. A 2013 RAND paper argued that joint aircraft programs historically experienced higher acquisition cost growth over single-service aircraft programs and did not save costs over the program life cycle. (Lorell, 2013) Further analysis of that assertion is beyond the scope of this paper.

rapid return to the underlying curve. The impact of building multiple models, by contrast, can extend across the entire program life cycle.

Theoretically, the degree of learning loss should vary proportionally to the extent of the configuration differences between the different variants. How much learning we can expect to be shared or lost between variants will depend on several factors. For example, for an aircraft program we might ask:

1. How common are the airframe engineering designs between the different variants?
2. Do they use a common set of mission and vehicle systems?
3. Will the different variants be built on a common production line, or will they be built on separate production lines, possibly even by different companies?
4. To what extent will the different variants be built using common tooling or manufacturing processes?
5. Will each variant be built using dedicated crews of assemblers? Or will crews be cycled between models as aircraft move down the production line?

The answers to these questions will determine how much learning transfer between variants we might reasonably expect. They will also inform us which estimating methodology would serve us best.

Jones (2019) identifies four estimating options for dealing with commonality. We will deal with each of these in turn in the following sections:

1. Ignore Differences (ID) – Assume a common learning curve and ignore any cost impact of multiple models.
2. Fixed Factors (FF) – Assume a common underlying curve and adjust for variant differences through a fixed factor or relationship between variants.
3. Total Separation (TS) – Assume each variant has a unique learning curve slope and that no learning transfer occurs between variants.
4. Proportional Representation (PR) – Assume a given combination of common or unique work has its own peculiar learning curve, but all of them share a common rate of learning.

To this list, we will add a fifth alternative:

5. Partial Separation (PS) – Assume each variant has a unique learning curve slope but allow that there is learning transfer between variants.

For simplicity of reference, this paper will also use Jones' nomenclature to refer to each of these options.

Three of these five options assume that the multiple variants will share at least a common rate of learning. Why might we expect different variants to share common learning curves? Multiple variants in a single program may nonetheless share many common decisions -- such as common investment strategies, design tools, tooling and build philosophy. A 1981 study of the contributors to learning reported that 78% of cost improvement could be attributed to causes other than operator learning (tooling, design engineering, management controls). (Jefferson, 1981) And even the mechanics themselves should be able to continue learning on similar if not identical operations, provided their crews are permitted to cycle between model versions. This argues that the observed rate of learning on a multi-variant program will tend to be more common than unique.

A Notional Program

To illustrate application of these methodologies but avoid compromising proprietary information, a dummy set of data has been constructed. This notional data presumes a two variant aircraft program with 80% of the production aircraft built in an U.S. Air Force (USAF) configuration while the remaining 20% are a U.S. Navy (USN) configuration. We will designate the USAF aircraft as Model A and the USN aircraft as Model B. A total of eight Engineering and Manufacturing Development (EMD) aircraft and 500+ production aircraft are built. For this analysis, we will assume that 65% of the manufacturing effort is common between the USAF and USN versions, with the remaining 35% being unique to each variant.

Figure 1 displays a subset of this database:

Total	Phase	Service	Variant	Adj Hours
1	EMD	USAF	A	384,354
2	EMD	USAF	A	392,722
3	EMD	Navy	B	359,041
4	EMD	Navy	B	366,820
5	EMD	USAF	A	316,530
6	EMD	USAF	A	303,031
7	EMD	Navy	B	355,896
8	EMD	Navy	B	329,786
9	Lot 1	USAF	A	294,270
10	Lot 1	USAF	A	283,824
	⋮			⋮
505	Lot 14	USAF	A	62,361
506	Lot 14	USAF	A	62,911
507	Lot 14	USAF	A	61,762
508	Lot 14	Navy	B	73,903
509	Lot 14	USAF	A	70,701
510	Lot 14	USAF	A	57,930
511	Lot 14	USAF	A	61,808
512	Lot 14	USAF	A	65,429
513	Lot 14	Navy	B	76,368
514	Lot 14	USAF	A	65,222

Figure 1. Subset of Notional Program Data

Figure 2 shows the program data graphically.

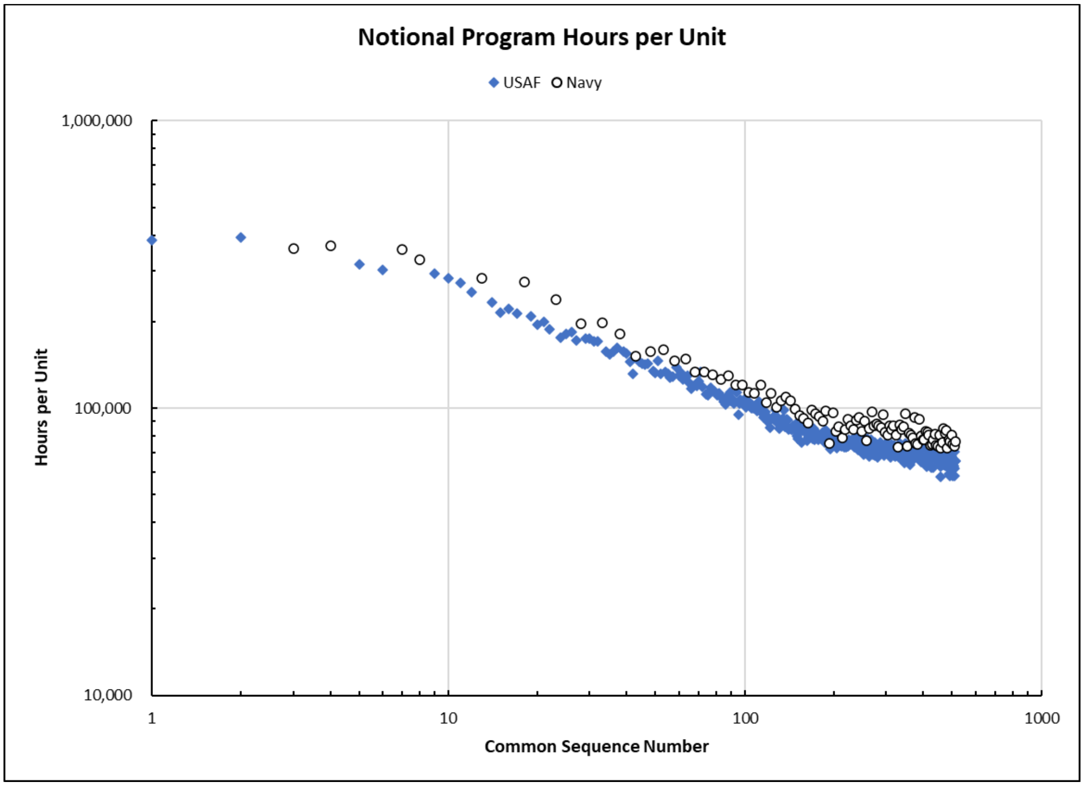

Figure 2. Notional Program Hours per Unit

The program data shows the familiar S-curve shape seen on many historical programs, where the development phase shows relatively shallow rates of learning, followed by an initial production phase where the rate of learning steepens and significant reductions in hours are made due to reduced engineering changes and improved processes and tooling. The third phase that follows displays a flattening of the learning curve slope as manufacturing processes stabilize and the program settles into higher production rates. (Engwall, 2001; Jones, 2001; Cochran, 1968; Cochran, 1961)

As demonstrated in prior articles (Johnstone, 2022), we can analyze data following the S-curve pattern using a piecewise regression. We start from our familiar improvement curve model:

$$y = \alpha_1 x^{\beta_1} \quad (1)$$

Where:

y = Manufacturing hours per unit
x = Cumulative units built to date
α_1 = Y-intercept, equal to theoretical first unit (TFU) hours
β_1 = Rate of learning, such that 2^β equals learning curve slope

After hours per unit and cumulative quantities are converted to natural logarithms, this yields the following linear form:

$$\ln y = \ln \alpha_1 + \beta_1 \ln x \quad (2)$$

Kennedy (1992) outlines a method for using dummy variables to capture a change in the intercept and slope coefficients between two periods. To create a two-leg segmented learning curve, we introduce breakpoint unit *T*. Based on our *a priori* selection for *T*, data is separated into pre-break period 1 ($x < T$) and post-break period 2 ($x \geq T$). In addition, dummy variable *D* is created such that *D* is zero for period 1, and one for period 2. Product dummy variable *Dx* is also created such that *Dx* takes the value *x* in period 2 but is 0 otherwise. This creates the regression equation:

$$\ln y = \ln \alpha_1 + \ln \alpha_2 D + \beta_1 \ln x + \beta_2 \ln Dx \quad (3)$$

Equation (3) represents two separate cases. Where $x < T$, variables *D* and *Dx* are 0 and equation (3) reduces back to our standard improvement curve equation (2). But where $x \geq T$ and *D* takes the value of one, different intercept and slope values are introduced such that:

$$\ln y = \ln(\alpha_1 + \alpha_2) + (\beta_1 + \beta_2) \ln Dx \quad (4)$$

Where:

- y = Manufacturing hours per unit (HPU)
- α_1 = Y-intercept for leg #1, equal to theoretical first unit hours for leg #1
- α_2 = Intercept adjustment for leg #2, such that $\alpha_1 + \alpha_2$ equals the Y-intercept for leg #2
- β_1 = Rate of learning for leg #1, such that 2^{β} equals learning curve slope #1
- β_2 = Rate of learning for leg #2, such that $2^{(\beta 1 - \beta 2)}$ equals learning curve for leg #2

Similarly, this methodology can be expanded to account for three or more legs of the learning curve.

We will use the first 370 aircraft (EMD and Lots 1-11) to develop historical learning curve slopes applying a piecewise regression as well as using each of these methodologies. We will then apply the resulting historical learning curves to forecast the next 144 aircraft (Lots 12-14). Comparing the forecast to the realized hours for those later aircraft will demonstrate some of the potential forecasting issues that may arise.

Ignore Differences (ID)

The first estimating option to deal with variant commonality is to simply ignore it. While this hardly seems like an option at all, there may be good reasons to consider it. Not all the differences between aircraft configurations result in a significant cost difference. For example, the Lockheed L-1011 commercial jetliner came in five unique configurations (-1, -100, -200, -250, -500) but from a learning curve perspective, only the shortened -500 configuration displayed a statistically significant variation in cost. The rest of the models could simply be combined for analysis purposes. (Benkard, 2000)

A visual review of Figure 2 tells us that there is a cost differential between the USAF and USN models, and that the ID methodology will probably not provide either a good fit to the data or provide particularly accurate forecasts. Nonetheless, we will set up our data for regression as follows under Figure 3. In each case we examine, the “common sequence number” (the actual build sequence, regardless of model) will stay the same. On the other hand, the “effective sequence number” will vary depending on the particular methodology we choose. For learning curve regression purposes, the effective sequence number will be the cumulative unit variable.

In the ID methodology, the effective sequence number is identical to the common sequence number. Thus, we are implicitly assuming that for cost purposes there is no difference between the variants and that 100% of the prior learning will be transferred between models. Note that there is nothing in the independent variables that identifies USAF or USN models.

				Curve Breakpoints		Dependent Variable	Independent Variables				
Common Sequence Number	Effective Sequence Number	Model	HPU	T_1	T_2	LN(HPU)	β1	α_2	β2	α_3	β3
1	1	A	384,354	9	151	12.86	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-
					⋮			⋮			
149	149	A	87,845	9	151	11.38	-	1	5.00	-	-
150	150	A	79,812	9	151	11.29	-	1	5.01	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04
					⋮			⋮			
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91

Figure 3. Subset of Notional Data Set Up for Regression (ID Methodology)

The results from the best fit regression are shown below in Figure 4:

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	LN_HRS = 12.89 + (-0.09531) * BETA1 + (-0.4265) * BETA2 + (-0.1968) * BETA3 + 0.6366 * ALPHA2 + (-0.5584) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8913	0.0601		214.6371	0.0000	1.0000
BETA1	-0.0953	0.0406	-0.0559	-2.3484	0.0193	0.9807
BETA2	-0.4265	0.0092	-2.4114	-46.4237	0.0000	1.0000
BETA3	-0.1968	0.0200	-1.4561	-9.8498	0.0000	1.0000
ALPHA2	0.6366	0.0716	0.8415	8.8915	0.0000	1.0000
ALPHA3	-0.5584	0.1259	-0.7451	-4.4363	0.0000	1.0000

TFU - Leg 1	396,828
Slope - Leg 1	93.6%
Slope - Leg 2	74.4%
Slope - Leg 3	87.3%
TFU - Leg 2	750,051
TFU - Leg 3	227,039

Figure 4. Best Fit Regression – ID Methodology

Figure 5 shows the derived learning curve projected through the end of Lot 14. As expected, while the ID methodology provides a good forecast at the bottom-line, it does not compare well to Lot 12 and on actuals at the individual variant level. For instance, it does a poor job of forecasting the B model, which is understated by almost 13%.

However, if for a given estimate we are not concerned with accuracy at the variant level, the ID methodology may answer. In addition, where the two models differ only in a particular area, i.e., the cockpit for a one-seat versus a two-seat variant, the ID methodology will probably work for all the unaffected component areas.

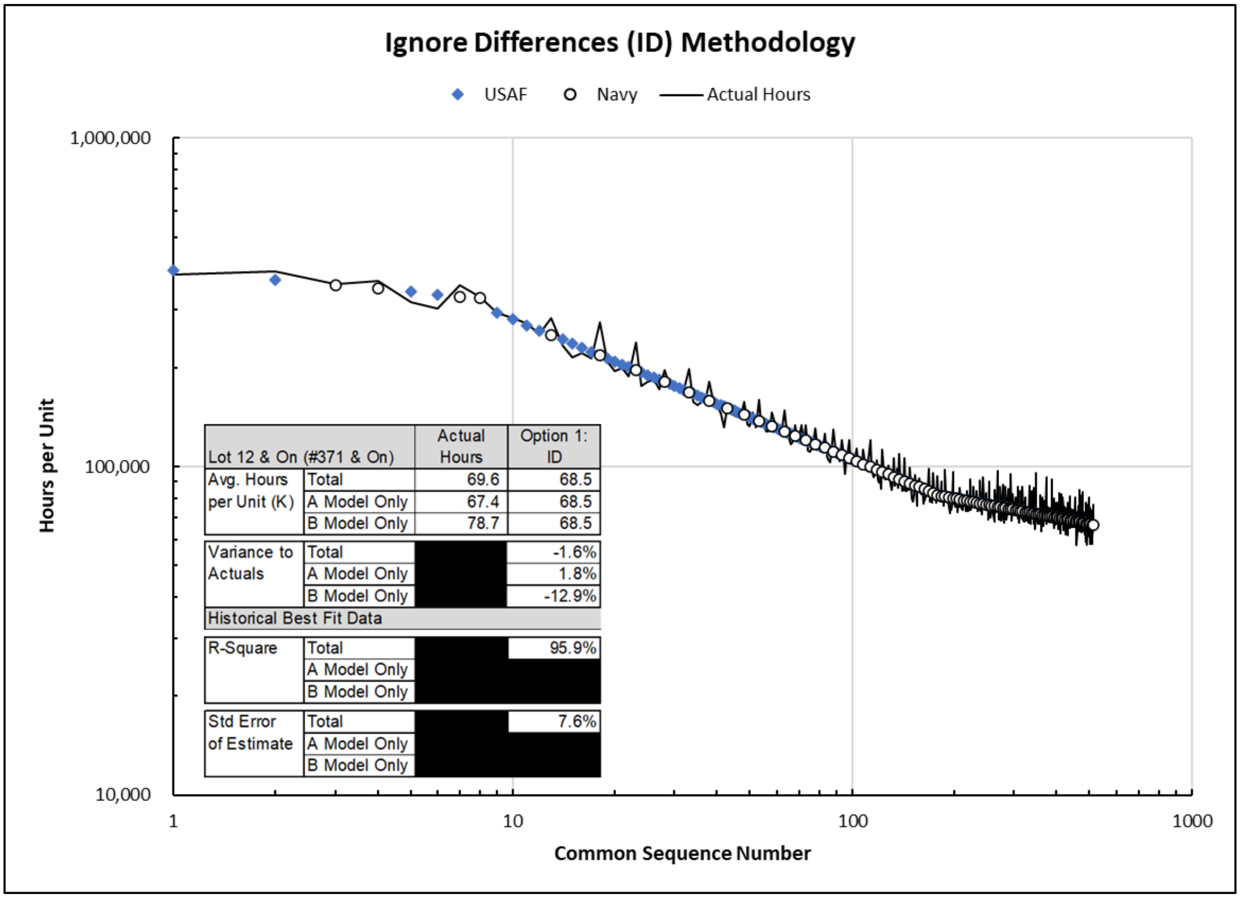

Figure 5. Learning Curve Best Fit & Forecast, ID Methodology

Fixed Factors (FF)

The Fixed Factors methodology is identical to the ID methodology, but with one critical difference: dummy variables (1 or 0) are established to account for the difference in aircraft models. Like the ID methodology, the effective sequence number is identical to the common sequence number and is not adjusted whenever there is a model change.

Figure 6 shows the setup of the data for regression, where we have added two columns at the far right of the table to add the B model dummy variable.

				Curve Breakpoints		Dependent Variable	Independent Variables					
Common Sequence Number	Effective Sequence Number	Model	HPU	T_1	T_2	LN(HPU)	β1	α_2	β2	α_3	β3	B Model Dummy
1	1	A	384,354	9	151	12.86	-	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-	1
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-	1
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-	1
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-	1
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-	-
					⋮			⋮				
149	149	A	87,845	9	151	11.38	-	1	5.00	-	-	-
150	150	A	79,812	9	151	11.29	-	1	5.01	-	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02	-
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02	-
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03	1
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04	-
					⋮			⋮				
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90	-
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91	-
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91	1
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91	-
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91	-

Figure 6. Subset of Notional Data Set Up for Regression (FF Methodology)

The results from the best fit regression are shown in Figure 7 below:

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	LN_HRS = 12.89 + (-0.1483) * BETA1 + (-0.4283) * BETA2 + (-0.1968) * BETA3 + 0.6199 * ALPHA2 + (-0.583) * ALPHA3 + 0.152 * B_MODEL

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8855	0.0351		366.9231	0.0000	1.0000
BETA1	-0.1483	0.0238	-0.0870	-6.2282	0.0000	1.0000
BETA2	-0.4283	0.0054	-2.4215	-79.7236	0.0000	1.0000
BETA3	-0.1968	0.0117	-1.4562	-16.8467	0.0000	1.0000
ALPHA2	0.6199	0.0419	0.8193	14.8043	0.0000	1.0000
ALPHA3	-0.5830	0.0736	-0.7780	-7.9213	0.0000	1.0000
B_MODEL	0.1520	0.0057	0.1669	26.4904	0.0000	1.0000

TFU - Leg 1	394,563
Slope - Leg 1	90.2%
Slope - Leg 2	74.3%
Slope - Leg 3	87.2%
TFU - Leg 2	733,354
TFU - Leg 3	220,252
B Model Factor	1.164

Figure 7. Best Fit Regression – FF Methodology

Interpretation of the B model dummy variable needs some explanation. The regression result (0.1520) is in logarithmic form and requires transformation. After exponentiation, the value becomes 1.164, which means that all else equal, the hours for the B variant are 16.4% higher than the A model.

Figure 8 shows the derived learning curve projected through the end of Lot 14. As expected, the FF forecast provides an improved fit against the Lot 12 and on actual hours both in total and at the variant level. It also visually displays that our use of a dummy variable for the USN model has essentially established two parallel learning curves with identical slopes – one for the A model and one for the B.

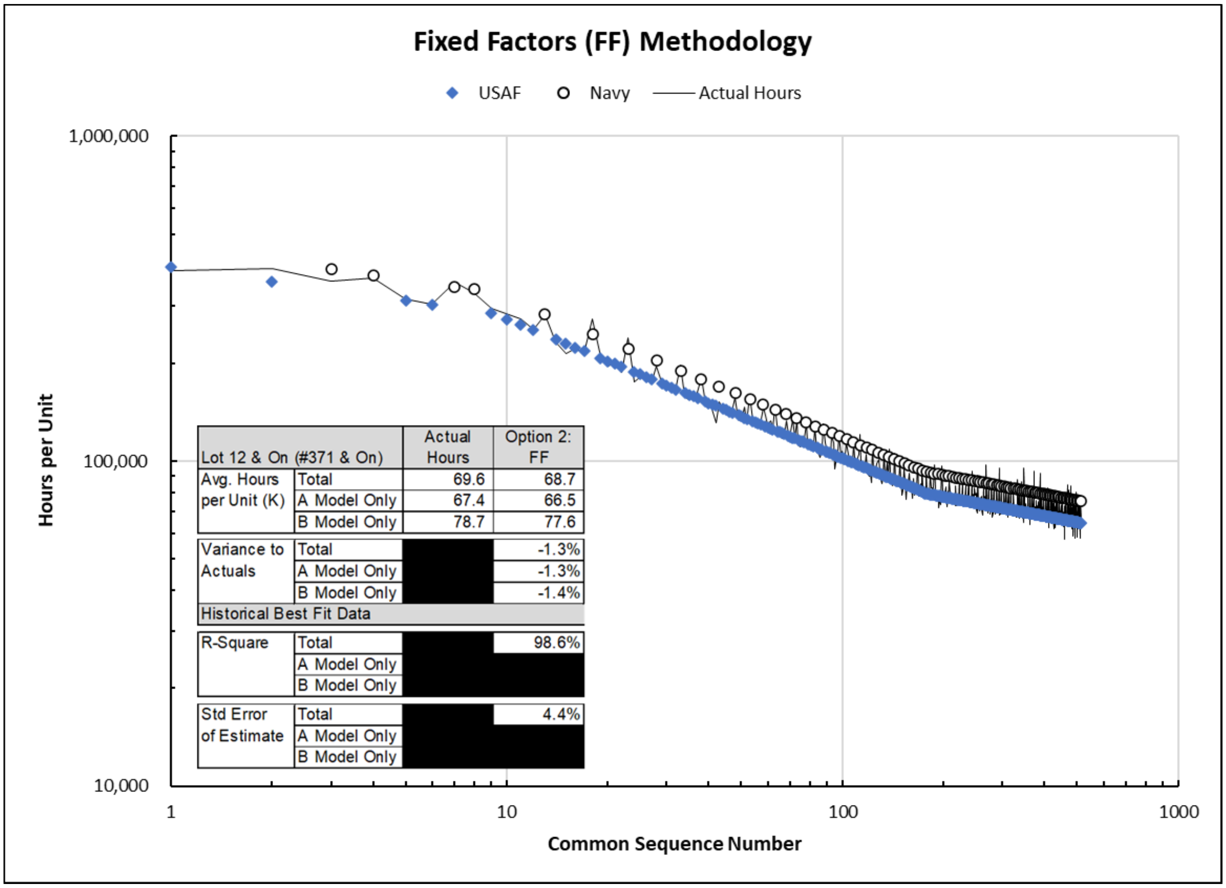

Figure 8. Learning Curve Best Fit & Forecast, FF Methodology

The FF methodology does make the implicit assumption that the cost relationship between the models is relatively stable over time. If we were forecasting hours without the benefit of prior actual hour history, the relationship between the variants could be established by other means, such as the use of Industrial Engineering standards.

Total Separation (TS)

The Total Separation (TS) methodology is the opposite of the ID methodology. Instead of assuming that all variants are common and that 100% learning transfer will take place between the variants, the TS methodology treats each variant as unique and assumes that *no* learning transfer will take place.

The TS methodology is best suited for build environments where the individual models are produced in different locations. An excellent example is the Eurofighter Typhoon, which has four separate assembly lines (Germany, Italy, Spain, and the United Kingdom) where each participating country is responsible for final assembly of its national aircraft. (*Eurofighter Typhoon*, n.d.) It is also well suited for situations where unique work is performed, e.g., the cockpit for a one-seat fighter versus a two-seat trainer version. It might also make sense if each variant has a dedicated build crew, and there is no cycling of mechanics between models.

To prepare our data for regression analysis, the effective sequence numbers will be modified to count only the cumulative builds for each model, as shown in Figure 9. Note that the change in effective sequence methodology required us to adjust our breakpoints for the multi-leg curve, assuming we want to retain our previous breaks at the end of EMD (common sequence #8) and mid-Lot 7 (common sequence #150). In addition, we will separate our data into two distinct regression models, one for USAF and one for USN.

				Curve Breakpoints		Dependent Variable	Independent Variables				
Common Sequence Number	Effective Sequence Number	Model	HPU	T_1	T_2	LN(HPU)	β1	α_2	β2	α_3	β3
1	1	A	384,354	5	119	12.86	-	-	-	-	-
2	2	A	392,722	5	119	12.88	0.69	-	-	-	-
3	1	B	359,041	5	33	12.79	-	-	-	-	-
4	2	B	366,820	5	33	12.81	0.69	-	-	-	-
5	3	A	316,530	5	119	12.67	1.10	-	-	-	-
6	4	A	303,031	5	119	12.62	1.39	-	-	-	-
7	3	B	355,896	5	33	12.78	1.10	-	-	-	-
8	4	B	329,786	5	33	12.71	1.39	-	-	-	-
9	5	A	294,270	5	119	12.59	-	1	1.61	-	-
10	6	A	283,824	5	119	12.56	-	1	1.79	-	-
					⋮			⋮			
149	117	A	87,845	5	119	11.38	-	1	4.76	-	-
150	118	A	79,812	5	119	11.29	-	1	4.77	-	-
151	119	A	78,318	5	119	11.27	-	-	-	1	4.78
152	120	A	81,745	5	119	11.31	-	-	-	1	4.79
153	33	B	94,523	5	33	11.46	-	-	-	1	3.50
154	121	A	86,816	5	119	11.37	-	-	-	1	4.80
					⋮			⋮			
366	291	A	66,039	5	119	11.10	-	-	-	1	5.67
367	292	A	66,241	5	119	11.10	-	-	-	1	5.68
368	76	B	78,852	5	33	11.28	-	-	-	1	4.33
369	293	A	72,902	5	119	11.20	-	-	-	1	5.68
370	294	A	71,358	5	119	11.18	-	-	-	1	5.68

Figure 9. Subset of Notional Data Set Up for Regression (TS Methodology)

The results from the regression are shown in Figure 10 (USAF model only) and Figure 11 (USN model only).

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	294
Equation in Unit Space:	LN_HRS = 12.9 + (-0.1862) * BETA1 + (-0.3977) * BETA2 + (-0.209) * BETA3 + 0.3635 * ALPHA2 + (-0.5838) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero		
Intercept	12.9047	0.0358		360.3412	0.0000	1.0000	TFU - Leg 1	402,196
BETA1	-0.1862	0.0377	-0.0589	-4.9395	0.0000	1.0000	Slope - Leg 1	87.9%
BETA2	-0.3977	0.0049	-2.2356	-80.9902	0.0000	1.0000	Slope - Leg 2	75.9%
BETA3	-0.2090	0.0115	-1.5626	-18.1867	0.0000	1.0000	Slope - Leg 3	86.5%
ALPHA2	0.3635	0.0408	0.5084	8.9133	0.0000	1.0000	TFU - Leg 2	578,528
ALPHA3	-0.5838	0.0707	-0.8213	-8.2568	0.0000	1.0000	TFU - Leg 3	224,336

Figure 10. Best Fit Regression – TS Methodology (USAF Only)

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	76
Equation in Unit Space:	LN_HRS = 12.81 + (-0.05086) * BETA1 + (-0.5459) * BETA2 + (-0.1468) * BETA3 + 0.5661 * ALPHA2 + (-0.8621) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero		
Intercept	12.8135	0.0579		221.4691	0.0000	1.0000	TFU - Leg 1	367,146
BETA1	-0.0509	0.0609	-0.0272	-0.8351	0.4065	0.5935	Slope - Leg 1	96.5%
BETA2	-0.5459	0.0231	-1.8901	-23.6604	0.0000	1.0000	Slope - Leg 2	68.5%
BETA3	-0.1468	0.0394	-0.7215	-3.7246	0.0004	0.9996	Slope - Leg 3	90.3%
ALPHA2	0.5661	0.0875	0.6818	6.4670	0.0000	1.0000	TFU - Leg 2	646,678
ALPHA3	-0.8621	0.1671	-1.0629	-5.1601	0.0000	1.0000	TFU - Leg 3	155,033

Figure 11. Best Fit Regression – TS Methodology (USN Only)

Comparison of the derived learning curve slopes shows a substantive difference between the two variants now. The B model EMD slope is 96.5% versus 87.9% for the A model, while the middle leg has a 68.5% slope for the B model versus 75.9% for the A model. The slope for the middle leg is so steep for the B model because the same reduction in B model hours is now calculated over B peculiar units 5 thru 32, not common units 9 thru 150 as in the previous examples. If in fact there is significant cross-variant learning benefit occurring, the TS methodology has in this instance overstated the true rate of B model learning.

Conversely, Jones (2019) argues that over longer production runs, the TS methodology is likely to produce *inflated* values. Notice for instance the significant difference between slopes for the third leg – 90.3% for B model versus 86.5% for A model – if this was extended out another several hundred units, we might see such an overstatement.

Figure 12 shows the derived learning curve projected through the end of Lot 14. The TS forecast provides the tightest fit to the Lot 12 and on actual hours. This is more a function of luck than skill, though, because there is a wider gap at the variant level, particularly for the B model.

Regardless of fit statistics, however, the TS methodology would only be appropriate if we are reasonably certain there is limited or no transfer of learning between variants. To use it otherwise would risk the potential understatement or overstatement of future hours.

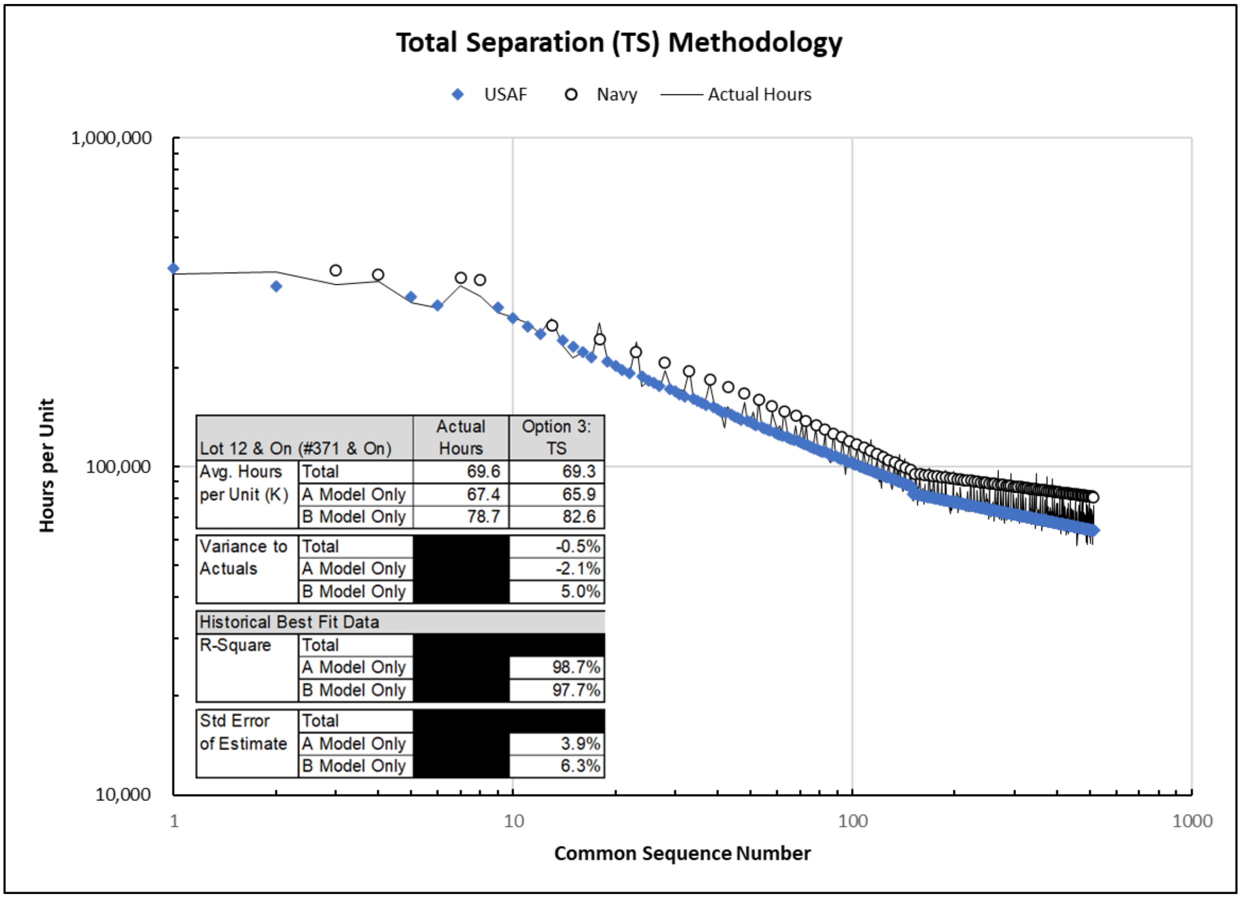

Figure 12. Learning Curve Best Fit & Forecast, TS Methodology

Partial Separation (PS)

The Partial Separation (PS) methodology is a hybrid methodology. Like the TS approach, it calculates different rates of learning for each variant. Like FF, it assumes there is learning transfer between variants.

The only differences in the setup of our data compared to the TS methodology is that we will revert to making the effective sequence number equal to the common sequence number and reset our curve breakpoints accordingly before splitting our data into separate USAF and USN runs, as seen in Figure 13:

Common Sequence Number	Effective Sequence Number	Model	HPU	Curve Breakpoints		Dependent Variable	Independent Variables				
				T_1	T_2	LN(HPU)	β1	α_2	β2	α_3	β3
1	1	A	384,354	9	151	12.86	-	-	-	-	-
2	2	A	392,722	9	151	12.88	0.69	-	-	-	-
3	3	B	359,041	9	151	12.79	1.10	-	-	-	-
4	4	B	366,820	9	151	12.81	1.39	-	-	-	-
5	5	A	316,530	9	151	12.67	1.61	-	-	-	-
6	6	A	303,031	9	151	12.62	1.79	-	-	-	-
7	7	B	355,896	9	151	12.78	1.95	-	-	-	-
8	8	B	329,786	9	151	12.71	2.08	-	-	-	-
9	9	A	294,270	9	151	12.59	-	1	2.20	-	-
10	10	A	283,824	9	151	12.56	-	1	2.30	-	-
					⋮			⋮			
149	149	A	87,845	9	151	11.38	-	1	5.00	-	-
150	150	A	79,812	9	151	11.29	-	1	5.01	-	-
151	151	A	78,318	9	151	11.27	-	-	-	1	5.02
152	152	A	81,745	9	151	11.31	-	-	-	1	5.02
153	153	B	94,523	9	151	11.46	-	-	-	1	5.03
154	154	A	86,816	9	151	11.37	-	-	-	1	5.04
					⋮			⋮			
366	366	A	66,039	9	151	11.10	-	-	-	1	5.90
367	367	A	66,241	9	151	11.10	-	-	-	1	5.91
368	368	B	78,852	9	151	11.28	-	-	-	1	5.91
369	369	A	72,902	9	151	11.20	-	-	-	1	5.91
370	370	A	71,358	9	151	11.18	-	-	-	1	5.91

Figure 13. Subset of Notional Data Set Up for Regression (PS Methodology)

The results from the regression are shown in Figure 14 (USAF model only) and Figure 15 (USN model only).

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	294
Equation in Unit Space:	LN_HRS = 12.91 + (-0.1452) * BETA1 + (-0.4264) * BETA2 + (-0.211) * BETA3 + 0.5924 * ALPHA2 + (-0.5247) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.9053	0.0335		384.8929	0.0000	1.0000
BETA1	-0.1452	0.0268	-0.0606	-5.4261	0.0000	1.0000
BETA2	-0.4264	0.0052	-2.5485	-82.2160	0.0000	1.0000
BETA3	-0.2110	0.0114	-1.6463	-18.4469	0.0000	1.0000
ALPHA2	0.5924	0.0401	0.8284	14.7761	0.0000	1.0000
ALPHA3	-0.5247	0.0717	-0.7383	-7.3223	0.0000	1.0000

TFU - Leg 1	402,459
Slope - Leg 1	90.4%
Slope - Leg 2	74.4%
Slope - Leg 3	86.4%
TFU - Leg 2	727,760
TFU - Leg 3	238,136

Figure 14. Best Fit Regression – PS Methodology (USAF Only)

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	76
Equation in Unit Space:	LN_HRS = 12.89 + (-0.07207) * BETA1 + (-0.437) * BETA2 + (-0.1398) * BETA3 + 0.8028 * ALPHA2 + (-0.7484) * ALPHA3

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8904	0.1263		102.0407	0.0000	1.0000
BETA1	-0.0721	0.0754	-0.0675	-0.9563	0.3422	0.6578
BETA2	-0.4370	0.0175	-2.2601	-24.9591	0.0000	1.0000
BETA3	-0.1398	0.0358	-0.9557	-3.9088	0.0002	0.9998
ALPHA2	0.8028	0.1467	0.9670	5.4710	0.0000	1.0000
ALPHA3	-0.7484	0.2349	-0.9227	-3.1861	0.0021	0.9979

TFU - Leg 1	396,487
Slope - Leg 1	95.1%
Slope - Leg 2	73.9%
Slope - Leg 3	90.8%
TFU - Leg 2	884,908
TFU - Leg 3	187,584

Figure 15. Best Fit Regression – PS Methodology (USN Only)

Comparing the regression results from our PS and TS runs reveals some interesting contrasts. In particular, the learning slopes for the PS case are all shallower than in the TS case. This is most evident for the USN model, particularly noticeable for its second leg which has flattened from 68.5% to 73.9% and is which within range of the USAF slope (74.4%) over the same range. The USAF slopes are flatter as well, although these changes are on a smaller scale.

Figure 16 shows the derived learning curve projected through the end of Lot 14. The PS method provides a tighter forecast for the B model than does the TS model, while the A model forecast shows the same variance as observed before.

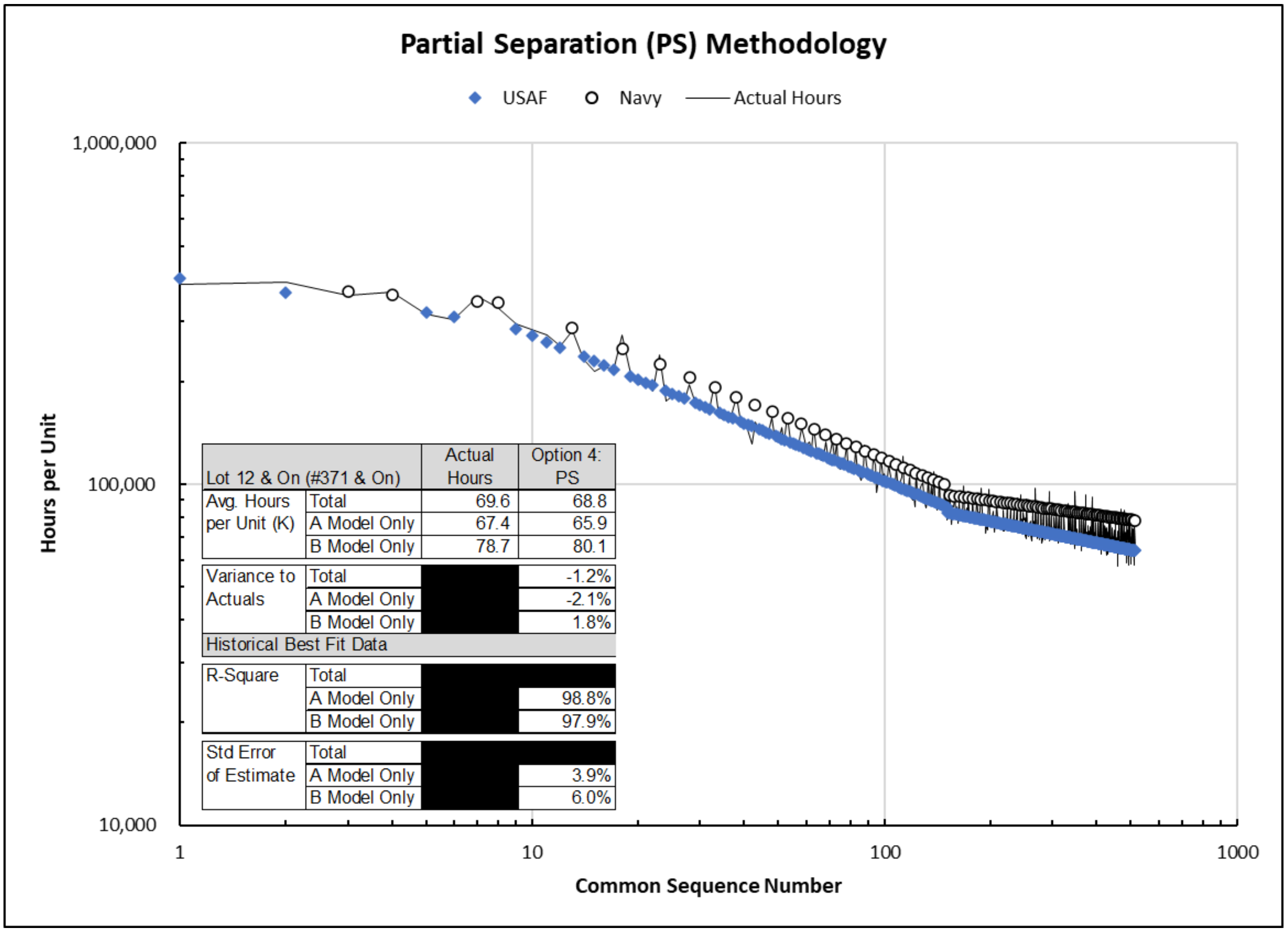

Figure 16. Learning Curve Best Fit & Forecast, PS Methodology

Proportional Representation (PR)

The proportional representation methodology provides a different methodology for calculating learning curve cumulative units. In lieu of using all units or only the units of a singular variant, the PR methodology derives a weighted calculation based on the ratio of common to unique work.

We will briefly take a break from the two-variant notional program we have been examining to explore some of the complexities for a three (or more) variant program. It is for such situations that the PR approach was initially developed. (Garg, 1961)

If we have three variants (A, B and C models), there are seven (7) possible combinations of common and unique work. (During the early days of F-35 development, we joked with our Northrop Grumman and BAE Systems counterparts about the "seven flavors of commonality" -- hence, the title of this paper.) We can have ABC common – that is, work that is common to all three variants. Likewise, we can have AB common – work common to the A and B variants, but which is not common to the C model. Similarly, there is AC common and BC common work. Lastly, there is work that is peculiar to a single variant and has no equivalent (A unique, B unique, C unique).

In general, we can calculate the number of combinations using the formula $2^x - 1$ where *x* is the number of variants, as illustrated in Figure 17.

Variants	Number
2	3
3	7
4	15
5	31
6	63

Figure 17. Number of Possible Combinations

For a given aircraft or component, how do we calculate the percentage of work that is common and unique? There is no consensus how to do this. Zhang (2019) suggests no less than seven different measures of commonality suggested by previous studies. In the author's experience, he has seen the following approaches suggested:

- Count number of common vs unique engineering drawings (Garg, 1961).
- Count number of common vs unique parts.
- Sum the empty weight of common vs unique parts.
- Sum the Industrial Engineering standard hours of common vs unique parts.
- Engineering judgment based on the similarity or uniqueness of assembly processes and tooling.

An additional complication is that a given part may be highly similar between two or more variants, but not identical. It follows parts or components that are similar should have some degree of learning transfer between variants. To address this the JAST program, which eventually evolved into the F-35, created the following definitions:

- Common: Physically identical and interchangeable
- Cousin: Same material, function, and interfaces – similar internal geometry, e.g., bulkheads made of identical material, same external dimensions, yet different web thickness and number of penetrations). Made using common fabrication or assembly tooling.
- Unique: Single variant application. (*JAST Commonality Study*, 1996)

Using the empty weight methodology outlined above, the weight of a cousin part was allocated 85% to the common category and 15% to unique. This was done on the assumption that a cousin part would retain most, but not all, of the cost advantages of a common part. (*JAST Commonality Study*, 1996) Imagine two composite skins made from the same graphite material, laid up on a common tool, and sharing the same outer mold line, but with different ply buildups to account for different load patterns. The common characteristics of these parts were judged to be sufficient to allow a high degree of learning transfer between the two versions of the skin.

In the end, the goal is to construct a table like Figure 18, which shows a theoretical example for a product with three variants:

	Percent Common to Each Model							
Model	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Total
A	50%	15%	10%		25%			100%
B	50%	15%		5%		30%		100%
C	50%		10%	5%			35%	100%

Figure 18. Notional Commonality Matrix

We can also construct a unit sequence table identifying the cumulative number of units which would be built under each commonality "flavor," as shown in Figure 19.

	Common/Unique Build Sequence Number						
Model	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique
A	1	1	1		1		
A	2	2	2		2		
B	3	3		1		1	
C	4		3	2			1
A	5	4	4		3		
A	6	5	5		4		
B	7	6		3		2	
C	8		6	4			2
A	9	7	7		5		
A	10	8	8		6		
B	11	9		5		3	
C	12		9	6			3

Figure 19. Cumulative Build Sequences (PR Methodology)

We can now apply a learning curve to each of these commonality combinations utilizing a single learning curve slope but using different theoretical first unit values for each variant (Jones, 2019; Garg, 1961). The results will be something like Figure 20.

Learning Curve Slope	85%
Learning Beta	-0.23447

		ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Variant Total
Work Content Split	A	50%	15%	10%		25%			100%
	B	50%	15%		5%		30%		100%
	C	50%		10%	5%			35%	100%
T-1 Hours	A	17,500	5,250	3,500	-	8,750	-	-	35,000
	B	22,500	6,750	-	2,250	-	13,500	-	45,000
	C	25,000	-	5,000	2,500	-	-	17,500	50,000

	Common/Unique Build Sequence Number							Hours per Unit							
Model	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Totals
A	1	1	1		1			17,500	5,250	3,500		8,750			35,000
A	2	2	2		2			14,875	4,463	2,975		7,438			29,750
B	3	3		1		1		17,391	5,217		2,250		13,500		38,358
C	4		3	2			1	18,063		3,865	2,125			17,500	41,552
A	5	4	4		3			11,999	3,793	2,529		6,763			25,084
A	6	5	5		4			11,497	3,600	2,400		6,322			23,819
B	7	6		3		2		14,257	4,435		1,739		11,475		31,906
C	8		6	4			2	15,353		3,285	1,806			14,875	35,319
A	9	7	7		5			10,454	3,327	2,218		6,000			21,999
A	10	8	8		6			10,199	3,224	2,149		5,749			21,322
B	11	9		5		3		12,824	4,032		1,543		10,434		28,833
C	12		9	6			3	13,961		2,987	1,642			13,526	32,116

Figure 20. Calculation of Three-Variant Model Using PR Methodology.

The results of such a calculation can be shown below in Figure 21, which shows the sawtooth pattern we have observed from our other commonality methodologies.:

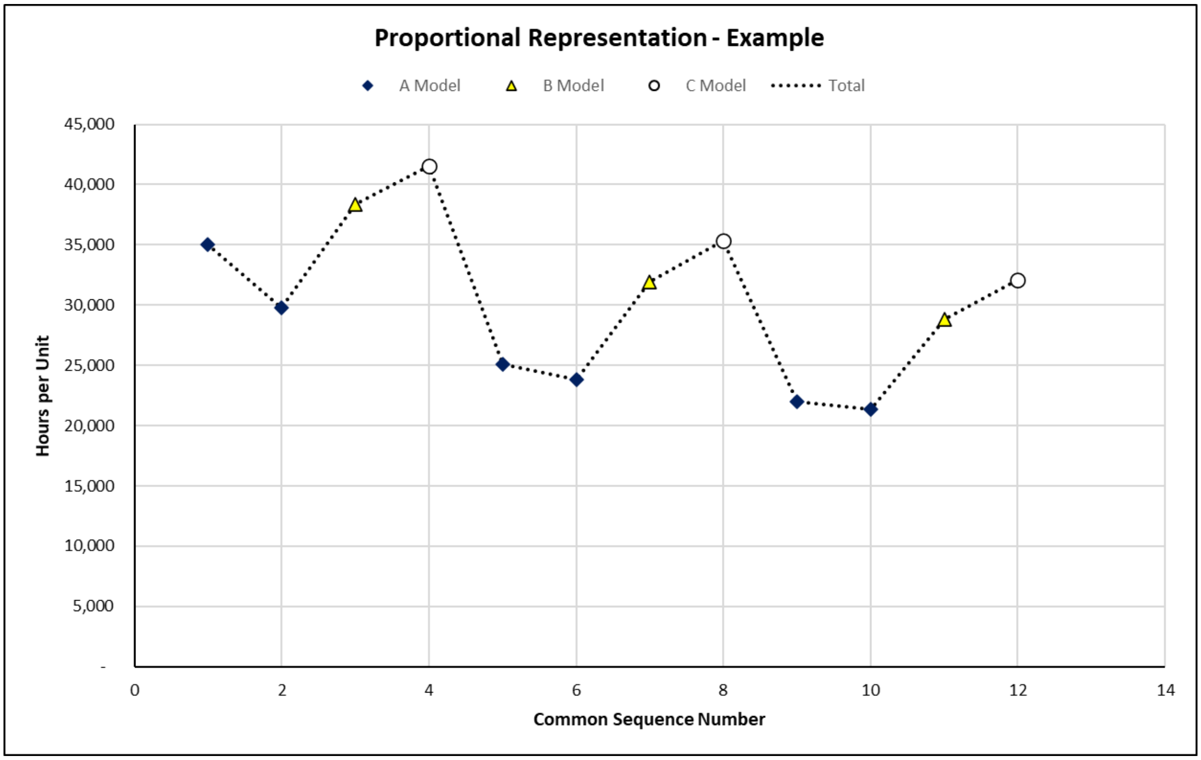

Figure 21. Results of the Three-Variant Model (PR Methodology)

The calculations, even in this simple three-variant version, can become quite involved. In addition, it is difficult to derive historical learning curve slopes from actual hours using this method. Jones (2019) suggests that Microsoft Excel Solver can be used to calculate theoretical first unit hours by variant and learning curve slope subject to certain constraints. However, as with any non-linear model, it is possible that Solver will not converge to a solution. In addition, using Solver means that the conventional regression statistics such as R-square or p values are not available. The calculations become even more complex when a multi-leg learning curve is introduced.

Fortunately, a simpler approach can be used to develop estimates as well as calculate historical performance.[2] It uses the percentages of common and unique work to calculate an effective sequence number, which will vary depending on which variant is being built. This allows the different variants to appear at separate points on the learning curve such that the hours per unit for more unique variants are calculated on earlier segments of the learning curve, resulting in higher hours.

Figure 22 shows the approach. Our commonality matrix shown in Figure 18 is combined such that, for each variant, the percentage learning credit incurred from the other models is calculated. This allows us to say that for each A model, not only does it earn a full unit of learning for each A model built, but it also receives 0.65 of a unit credit for each B model and 0.60 of a unit credit for each C model built to

[2] Acknowledgments to Kevin N. Curtis, retired Industrial Engineer and Lockheed Martin Fellow Emeritus, who showed me this approach some years ago.

that point. The specific amount of credit will depend on the ratio of common to unique work – as the percentage of unique work increase, the learning credit for building the other variants will decrease.

	Percent Common to Each Model							
Model	ABC Common	AB Common	AC Common	BC Common	A Unique	B Unique	C Unique	Total
A	50%	15%	10%		25%			100%
B	50%	15%		5%		30%		100%
C	50%		10%	5%			35%	100%

Commonality Matrix			
Model	A Credit	B Credit	C Credit
A	100%	65%	60%
B	65%	100%	55%
C	60%	55%	100%

For Every A Model Built:
Each A Model: A Receives 100% Learning Credit
Each B Model: ABC Common + AB Common = 50% + 15% = A Receives 65% Credit
Each C Model: ABC Common + AC Common = 50% + 10% = A Receives 60% Credit

	Cumulative Unit Count			Effective
Model	A	B	C	Unit
A	1	0	0	1.00
A	2	0	0	2.00
B	2	1	0	2.30
C	2	1	1	2.75
A	3	1	1	4.25
A	4	1	1	5.25
B	4	2	1	5.15
C	4	2	2	5.50
A	5	2	2	7.50
A	6	2	2	8.50
B	6	3	2	8.00
C	6	3	3	8.25

(3 A's x 100%) + (1 B x 65%) + (1 C x 60%) = Effective Unit 4.25

Figure 22. Alternate Methodology for Calculating Effective Unit Sequences

This methodology has the substantial advantage that it allows the historical learning curve slope to be easily calculated using a single effective unit value for each aircraft and permitting the use of conventional linear regression tools.

Back to our two-variant notional program. In this case, there are only three possible combinations (AB common, A unique, B unique). We will presume that 65% of the aircraft is common between the USAF and USN versions and the remaining 35% is variant-unique. Thus, we can develop our effective sequence table in Figure 23. Note as well that we have reintroduced our B model dummy variable that we used in the FF approach. We still need to account for the greater work content of the USN model independent of the learning curve impacts.

				Curve Breakpoints		Dependent Variable	Independent Variables					
Common Sequence Number	Effective Sequence Number	Model	HPU	T_1	T_2	LN(HPU)	β1	α_2	β2	α_3	β3	B Model Dummy
1	1.0	A	384,354	7.6	139.8	12.86	-	-	-	-	-	-
2	2.0	A	392,722	7.6	139.8	12.88	0.69	-	-	-	-	-
3	2.3	B	359,041	7.6	111.0	12.79	0.83	-	-	-	-	1
4	3.3	B	366,820	7.6	111.0	12.81	1.19	-	-	-	-	1
5	4.3	A	316,530	7.6	139.8	12.67	1.46	-	-	-	-	-
6	5.3	A	303,031	7.6	139.8	12.62	1.67	-	-	-	-	-
7	5.6	B	355,896	7.6	111.0	12.78	1.72	-	-	-	-	1
8	6.6	B	329,786	7.6	111.0	12.71	1.89	-	-	-	-	1
9	7.6	A	294,270	7.6	139.8	12.59	-	1	2.03	-	-	-
10	8.6	A	283,824	7.6	139.8	12.56	-	1	2.15	-	-	-
					⋮			⋮				
149	137.8	A	87,845	7.6	139.8	11.38	-	1	4.93	-	-	-
150	138.8	A	79,812	7.6	139.8	11.29	-	1	4.93	-	-	-
151	139.8	A	78,318	7.6	139.8	11.27	-	-	-	1	4.94	-
152	140.8	A	81,745	7.6	139.8	11.31	-	-	-	1	4.95	-
153	111.0	B	94,523	7.6	111.0	11.46	-	-	-	1	4.71	1
154	142.5	A	86,816	7.6	139.8	11.37	-	-	-	1	4.96	-
					⋮			⋮				
366	339.8	A	66,039	7.6	139.8	11.10	-	-	-	1	5.83	-
367	340.8	A	66,241	7.6	139.8	11.10	-	-	-	1	5.83	-
368	265.8	B	78,852	7.6	111.0	11.28	-	-	-	1	5.58	1
369	342.4	A	72,902	7.6	139.8	11.20	-	-	-	1	5.84	-
370	343.4	A	71,358	7.6	139.8	11.18	-	-	-	1	5.84	-

Figure 23. Subset of Notional Data Set Up for Regression (PR Methodology)

The results from the regression are shown in Figure 24.

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observations Used:	370
Equation in Unit Space:	LN_HRS = 12.88 + (-0.1342) * BETA1 + (-0.4215) * BETA2 + (-0.2076) * BETA3 + 0.554 * ALPHA2 + (-0.5313) * ALPHA3 + 0.08574 * B_MODEL

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	12.8807	0.0355		363.2449	0.0000	1.0000
BETA1	-0.1342	0.0268	-0.0704	-5.0024	0.0000	1.0000
BETA2	-0.4215	0.0054	-2.3133	-77.6938	0.0000	1.0000
BETA3	-0.2076	0.0116	-1.5020	-17.8864	0.0000	1.0000
ALPHA2	0.5540	0.0419	0.7322	13.2168	0.0000	1.0000
ALPHA3	-0.5313	0.0722	-0.7089	-7.3544	0.0000	1.0000
B_MODEL	0.0857	0.0061	0.0941	13.9869	0.0000	1.0000

TFU - Leg 1	392,667
Slope - Leg 1	91.1%
Slope - Leg 2	74.7%
Slope - Leg 3	86.6%
TFU - Leg 2	683,295
TFU - Leg 3	230,835
B Model Factor	1.090

Figure 24. Best Fit Regression – PR Methodology

Figure 25 shows the derived learning curve projected through the end of Lot 14. The PR method provides a reasonable forecast of Lot 12 and on with a slightly larger variance for the B model. But in this case the variance to actual hours is not quite as good as the FF method. It also has the disadvantage that, even with the simplified calculation of effective sequence numbers, of being the most computationally complex method.

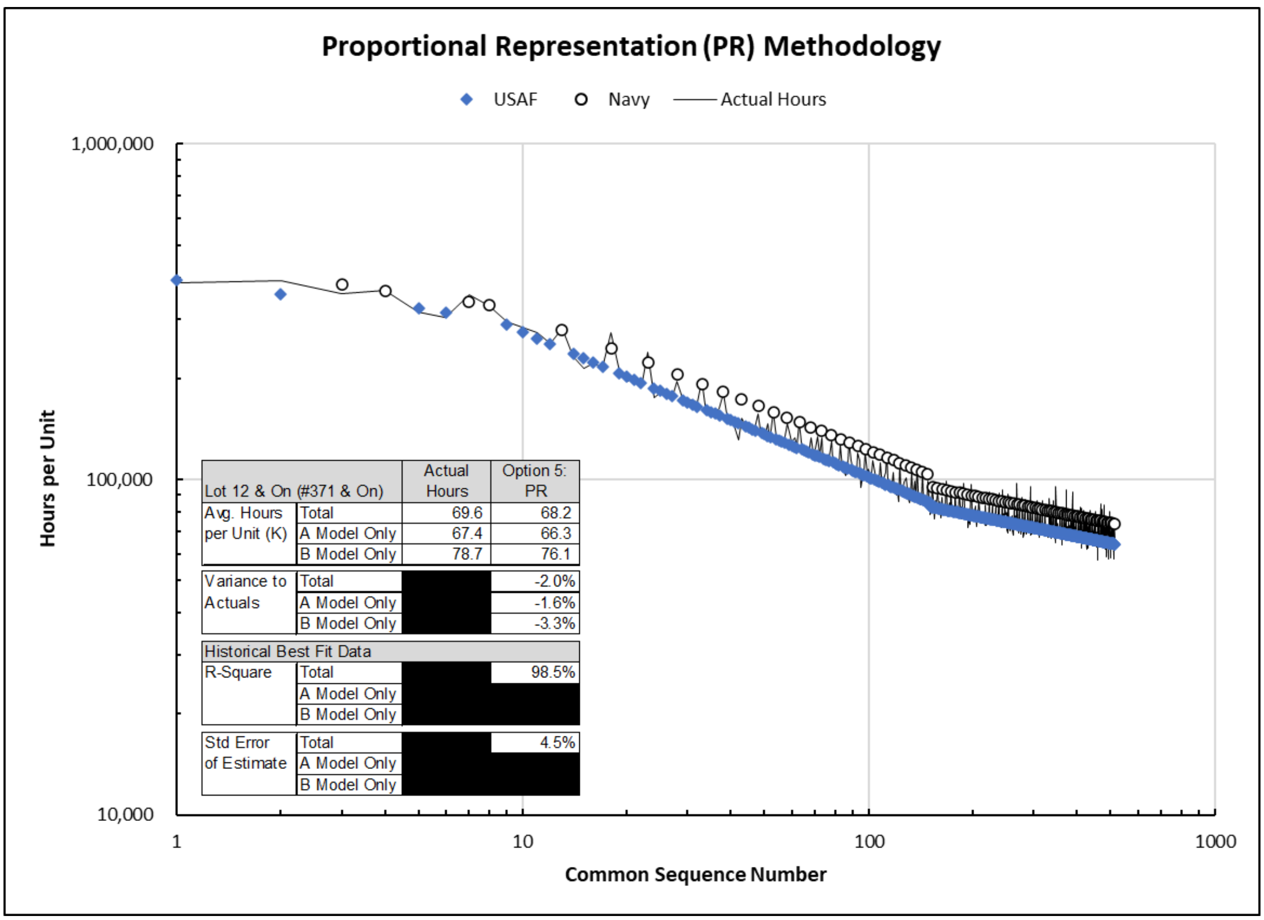

Figure 25. Learning Curve Best Fit & Forecast, PR Methodology

Conclusions

Figure 26 summarizes the five approaches to estimating commonality and shows how different the forward projections can be.

Lot 12 & On (#371 & On)		Actual Hours	Option 1: ID	Option 2: FF	Option 3: TS	Option 4: PS	Option 5: PR
Avg. Hours per Unit (K)	Total	69.6	68.5	68.7	69.3	68.8	68.2
	A Model Only	67.4	68.5	66.5	65.9	65.9	66.3
	B Model Only	78.7	68.5	77.6	82.6	80.1	76.1
Variance to Actuals	Total		-1.6%	-1.3%	-0.5%	-1.2%	-2.0%
	A Model Only		1.8%	-1.3%	-2.1%	-2.1%	-1.6%
	B Model Only		-12.9%	-1.4%	5.0%	1.8%	-3.3%
Historical Best Fit Data							
R-Square	Total		95.9%	98.6%			98.5%
	A Model Only				98.7%	98.8%	
	B Model Only				97.7%	97.9%	
Std Error of Estimate	Total		7.6%	4.4%			4.5%
	A Model Only				3.9%	3.9%	
	B Model Only				6.3%	6.0%	

Figure 26. Comparison of Forecasted Hours

It is worth nothing that from a best fit perspective, four of the five methods have a R-square value of greater than 98% with standard errors around 4%. From this perspective, only the ID method can be clearly rejected.

From the perspective of forecast accuracy, the differences between the methodologies become more apparent. Overall, the TS, FF and PS methodologies show the greatest accuracy at the top level, ranging from -0.5% to -1.3% error. However, the TS methodology shows a higher B model variance (+5.0%) than we would probably like. Of the remaining two methods (FF and PS), the FF option provides the forecast closer to the true actual hours for Lots 12 and on at the individual variant level (-1.3% versus -2.0% for A model and -1.4% versus +1.8% for B model).

This is hardly surprising because we “rigged” the notional data to make it so -- the hours per unit were generated using an FF approach before introducing a random error to provide a realistic spread of values. Had we generated the data using a different set of premises, another method would probably have produced the best forecast.

The purpose of this demonstration was not to provide proof that one method is always superior to the others. Its purpose is to pilot each method, and to show that the particulars of a program and its build circumstances will dictate which method is the preferred approach.

We might summarize the cases where each methodology might prove more appropriate below in Figure 27.

Methodology	More Appropriate If:	Less Appropriate If:
Ignore Differences ((D)	• There is little or no cost difference between variants.	• Significant differences in work content exist between variants.
Fixed Factors (FF)	• Significant amount of work is common or similar and the probability of learning transfer between variants is high. • The cost variance between models is expected to be a fixed ratio in the future, e.g., B models are 10% more costly than A models.	• If component or subcomponent is variant-unique (TS may be more appropriate for that item).
Total Separation (TS)	• Individual models are produced in different locations or on unique production lines, and the probability of learning transfer between variants is low. • A component or subcomponent is variant-unique (FF, PS or PR may be used for the other, more common build areas).	• Models are built in the same location and/or same production line with work crews being cycled between models.
Partial Separation (PS)	• Significant degree of common or similar work, but reason to believe each variant has a unique rate of learning.	• If the elements of learning that are common or similar between variants are high contributors to cost improvement, causing the rate of learning between variants to be roughly equal.
Proportional Representation (PR)	• Significant amount of work is common or similar and the probability of learning transfer between variants is high. • A fixed cost ratio between models cannot be established from actual cost history, or the relationship of one variant to another is expected to be different in the future.	• No suitable *a priori* methodology exists for determining the percentage of common vs unique work.

Figure 27. Comparison of Estimating Methodologies

As Figure 27 demonstrates, the appropriateness of using one methodology versus another is highly dependent on the program's peculiar circumstances.

In the introduction, we asked five questions:

1. How common are the airframe engineering designs between the different variants?
2. Do they use a common set of mission and vehicle systems?
3. Will the different variants be built on a common production line, or will they be built on separate production lines, possibly even by different companies?
4. To what extent will the different variants be built using common tooling or manufacturing processes?
5. Will each variant be built using dedicated crews of assemblers? Or will crews be cycled between models as aircraft move down the production line?

It is the answers to these questions – in addition to the traditional best fit regression statistics --that will provide the best guide to the estimator how to proceed.

References

Aboulafia, R. (1996). *Janes Civil Aircraft.* Glasgow, UK: HarperCollins.

About the Boeing 737 MAX (n.d.). Retrieved from https://www.boeing.com/commercial/737max/.

Benkard, C. L. (2000). Learning and Forgetting: The Dynamics of Aircraft Production. *American Economic Review,* 90(4), 1034-1054.

Cochran, E. B. (1960, July-August). New Concepts of the Learning Curve. *The Journal of Industrial Engineering*, 317-327.

Cochran, E. B. (1968). *Planning Production Costs: Using the Improvement Curve*. San Francisco, CA: Chandler Publishing Company.

Engwall, R. L. (2001). Learning Curves. In K.B. Zandin (Ed.), *Manyard's Handbook of Industrial Engineering* (pp. 17.83-17.104). New York, NY: McGraw Hill.

Eurofighter Typhoon. (n.d.). Retrieved from https://en.wikipedia.org/wiki/Eurofighter_Typhoon.

Garg, A., & Milliman, P. (1961, January-February). The Aircraft Progress Curve – Modified for Design Changes. *The Journal of Industrial Engineering*, 7(1), 23-28.

Jefferson, P. (1981, May) Productivity Comparisons with the USA – Where Do We Differ? *Aeronautical Journal*, 85(844), 179-184.

Johnstone, B.M. (2022, November). Projecting Future Costs with Improvement Curves: Perils and Pitfalls. *Journal of Cost Analysis and Parametrics*, 10(3), 64-82.

Johnstone, B. M. (2023, May). *Trouble with the Curve: Engineering Changes and Manufacturing Learning Curves.* 2023 International Cost Estimating and Analysis Association (ICEAA) Professional Development & Training Workshop, San Antonio, TX.

Joint Advanced Strike Technology (JAST) Commonality Study Final Report. (1996, August). Arlington, VA: Joint Advanced Strike Technology Program Office.

Jones, A. R. (2019). *Learning, Unlearning and Re-learning Curves*. London, UK: Routledge.

Jones, A.R. (2001). Case Study: Applying Learning Curves in Aircraft Production – Procedures and Experiences. In K.B. Zandin (Ed.), *Manyard's Handbook of Industrial Engineering* (pp. 17.197-17.213). New York, NY: McGraw Hill.

Kennedy, P. A. (1992). *A Guide to Econometrics* (3rd ed). Cambridge, MA: The MIT Press.

Lorell, M. A., Kennedy, M., Leonard, R. S., Munson, K., Abramson, S., An, R. A., Guffey, R. A. (2013) *Do Joint Fighter Programs Save Money?* MG-1225-AF. Santa Monica, CA: RAND.

Zhang, Y., Yang, Z., Ma, X., Dong, W., Dong, D., Tan, Z., Zhang, S. (2019) Exploration and Implementation of Commonality Valuation Method in Commercial Aircraft Family Design. *Chinese Journal of Aeronautics*, 32(8), 1828-1846. Retrieved from https://www.sciencedirect.com/science/ article/pii/ S100093611930216X.

Biography

Brent Johnstone is a Lockheed Martin Fellow and production air vehicle cost estimator at Lockheed Martin Aeronautics Company in Fort Worth, Texas. He has 36 years' experience in the military aircraft industry, including 33 years as a cost estimator. He has worked on the F-16 program and Advanced Development Programs and has been for 25 years the lead Production Operations cost estimator for the F-35 program. He has a Master of Science from Texas A&M University and a Bachelor of Arts from the University of Texas at Austin.

My appreciation to Bernie Belley, retired Director of F-35 Estimating & Pricing, who encouraged me to write and submit a paper on commonality.